

EPFL Cours d'Electrotechnique I

2. Conventions et les symboles:

→ concepts → modifs

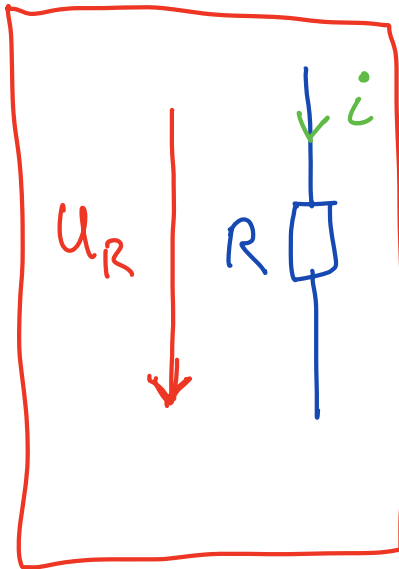
Ex : courant : i , I , $\hat{i}$, $\hat{I}$, $\bar{i}$, $\bar{I}$

unité : $[A]$

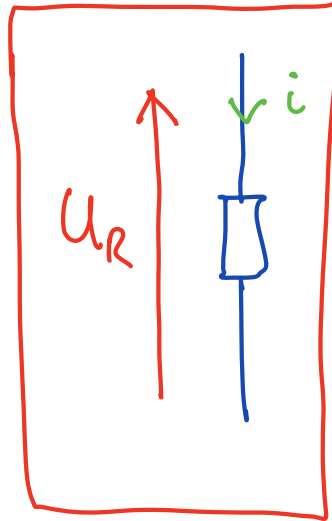
Relations : $u = R \cdot I$
 $u = R \cdot i$

Dessin : 
Résistance

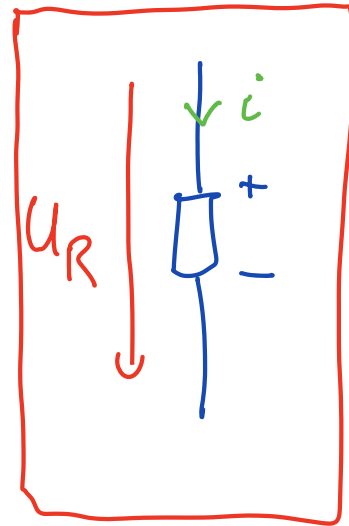
Choix à faire :



International



F_n

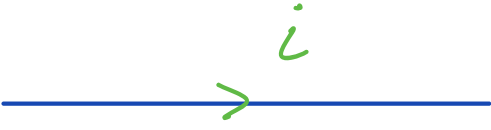


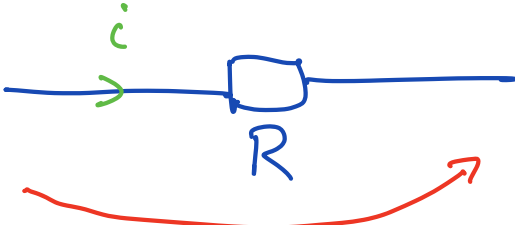
USA, B

Convention moteur : choix

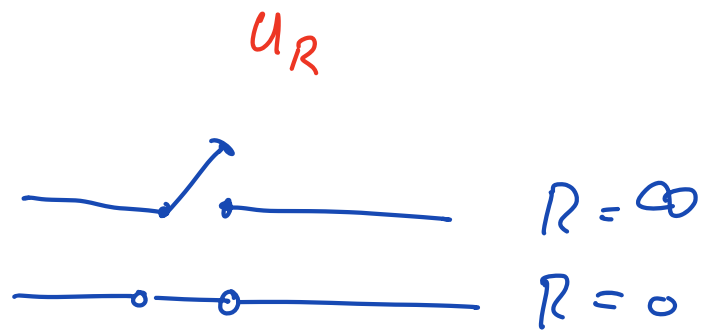
2.2 Représentation graphique :

Conducteur : 
parfait

Conducteur : 
avec un courant

Élément : 

Interrupteur :

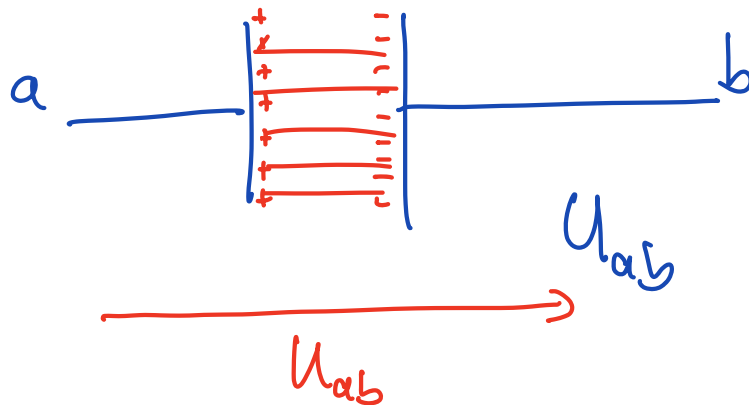


3. Lois fondamentales :

Différence de potentiel : Tension [Volt]

$$V_a - V_b = \int_a^b E dl = U_{ab} \quad [V]$$

$\xleftarrow{\quad l \quad} \xrightarrow{\quad}$



3.2.19 La Capacité :

Définition : Charge électrique : Q

Capacité :

$$C = \frac{Q}{U_{ab}}$$

Symbolle : 

3.3 Courant électrique :

$$I = \frac{dQ}{dt} \quad [A]$$

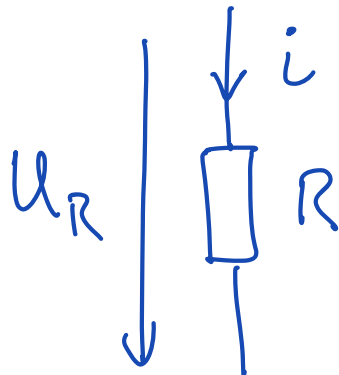
Densité de courant : $\vec{j} \Rightarrow [A/m^2]$

3.3.4 Pertes Joule :

$$P = R \cdot I^2 \quad [W]$$

Récap :

Convention Notion :



→ Puissance positive

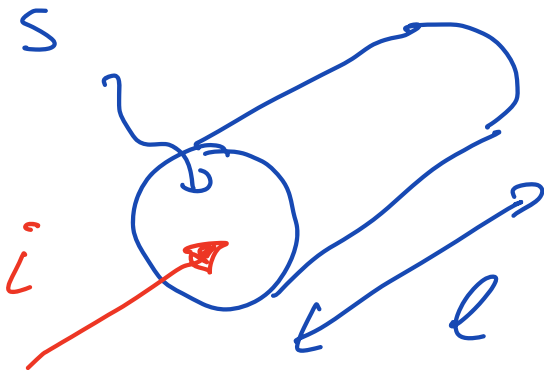
→ conv. Notion consommation.

3.3.6 Définition de la résistance :

$$R_{ab} = \int_a^b \rho \cdot \frac{dl}{S}$$

$\uparrow$ résistivité électrique $[\Omega m]$

$\leftarrow$ longueur
 $\nwarrow$ surface



Si S est constante sur la longueur

$$R_{ab} = \frac{\rho \cdot l}{S} \quad [\Omega]$$

3.3.8 Loi d'Ohm :

$$U_{ab} = R_{ab} I$$

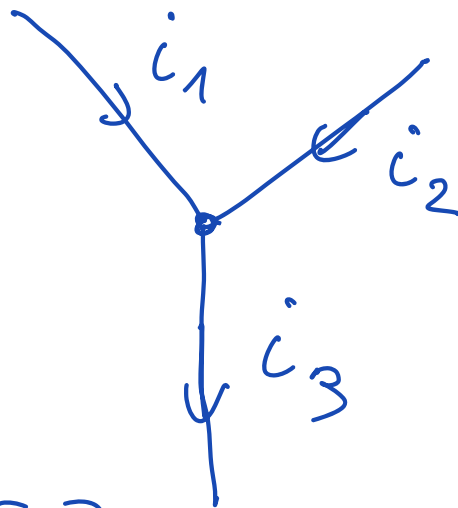
(courant et tension continue)

$$U_{ab} = R_{ab} \cdot i \quad (\text{courant et tension variable})$$

3.3.11 Lois de Kirchhoff :

Noeud : Point de convergence
d'au moins trois conducteurs

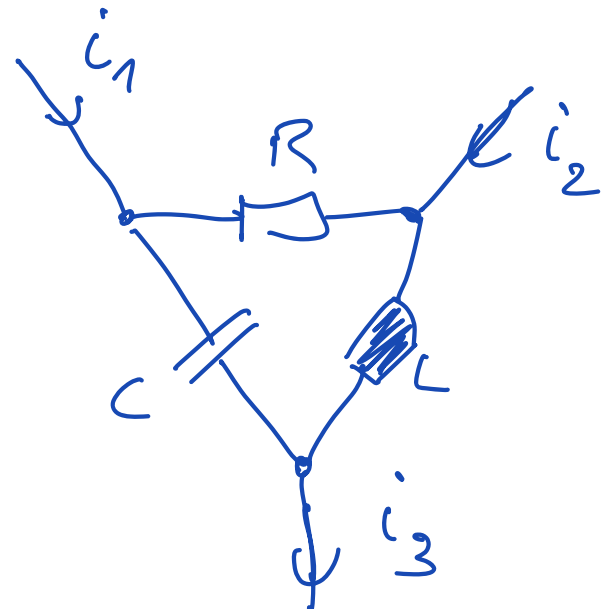
$$\underline{\sum i_j = 0}$$



$$i_1 + i_2 - i_3 = 0$$

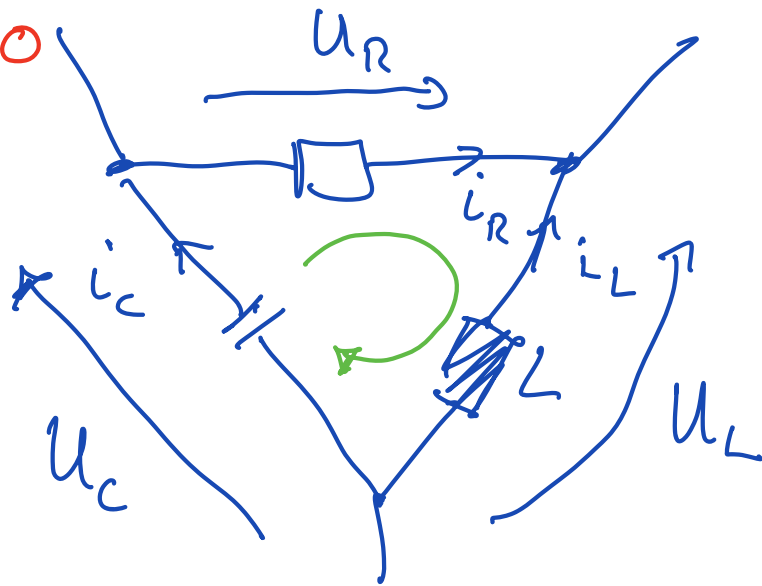
Noeud généralisé :

$$i_1 + i_2 - i_3 = 0$$



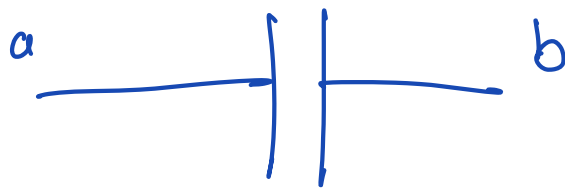
Maille : ensemble de branche partant d'un nœud pour y retourner

$$\sum U_j = 0$$



$$U_R - U_L + U_C = 0$$

3.5 La Capacité :

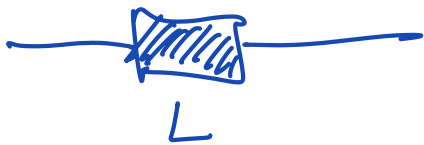


$$C = \frac{Q}{U_{ab}}$$

$$Q = \int i dt$$

$$u = \frac{1}{C} \int i dt$$

3.4 l'inductance :



$$u = L \frac{di}{dt}$$

$$\begin{cases} \vec{\text{Rot}} \vec{H} = \vec{J} \\ \vec{\text{Rot}} \vec{E} = -\frac{d\vec{B}}{dt} \end{cases}$$

$L [H]$ Henry

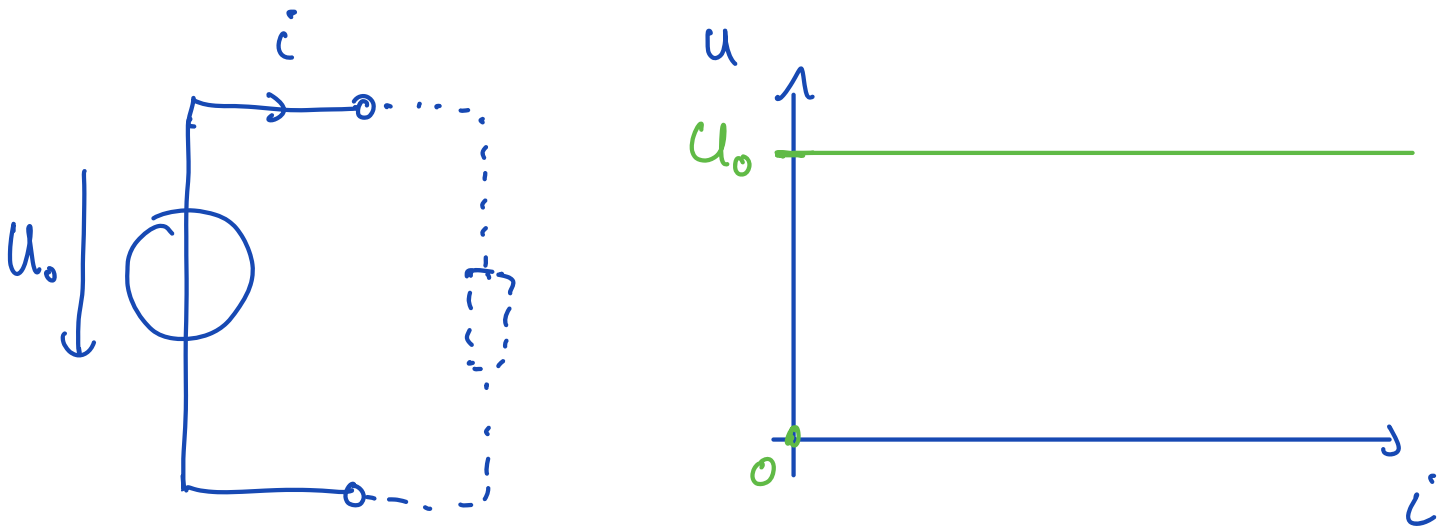
4. Eléments de circuit :

4.1 Dipôle : circuit qui possède 2 bornes



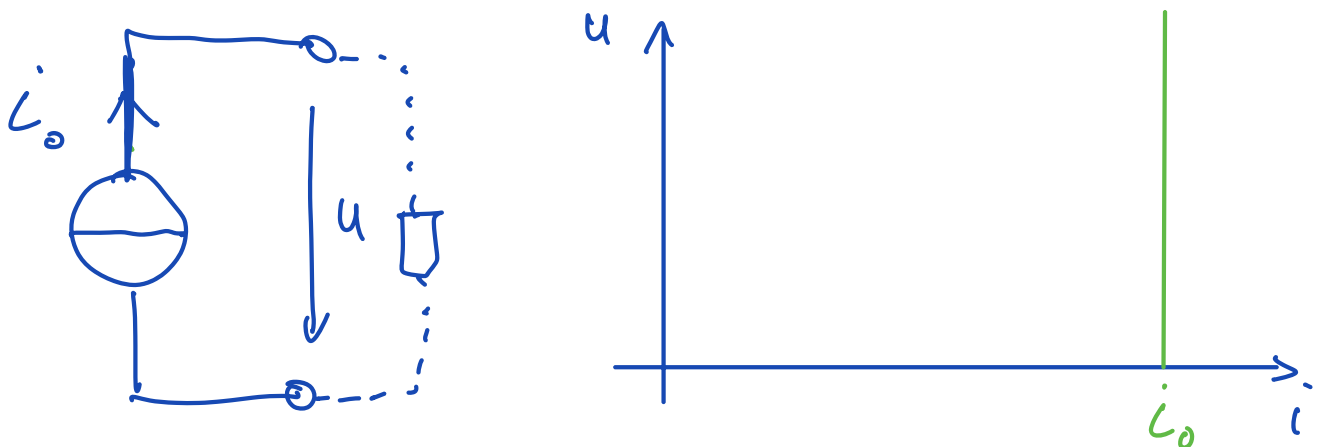
4.2 Sources de tension et de courant

a) Source de tension idéale :



c'est un élément virtuel, idéal et inexistant dans la nature

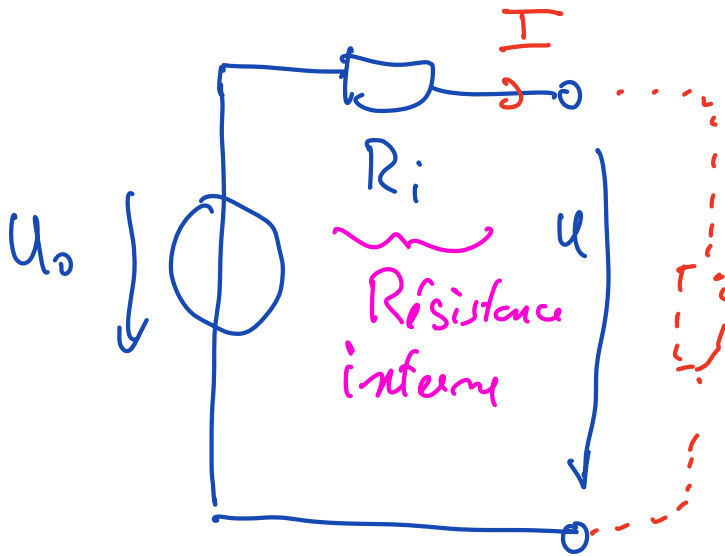
b) Source de courant idéale :



élément virtuel, inexistant dans la nature.

4.2.5 Source de tension réelle :

Def :



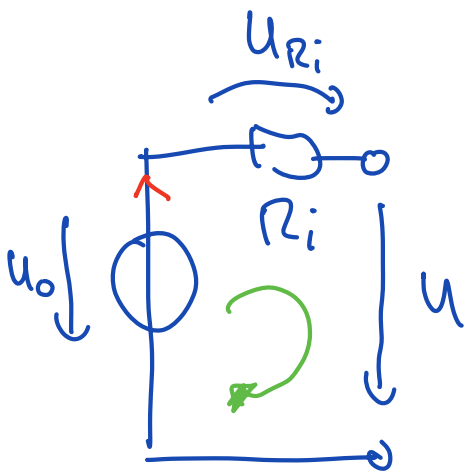
S. Tension
idéale

S. tension réelle

U_0 : Tension de
la source idéale
Tension à vide

R_i : Résistance
interne

U : Tension de
la source

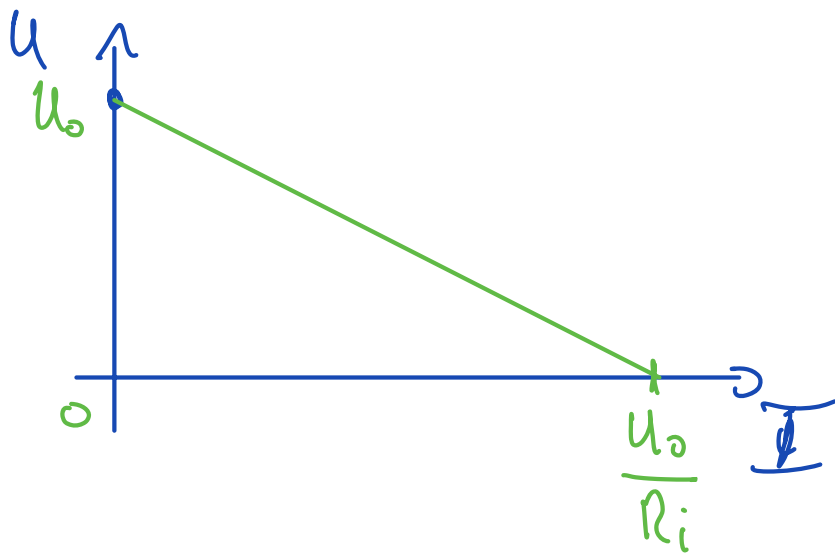


$$\sum U = 0$$

$$-U_0 + U_{R_i} + U = 0$$

$= R_i \cdot I$

$$U = U_0 - R_i \cdot I$$



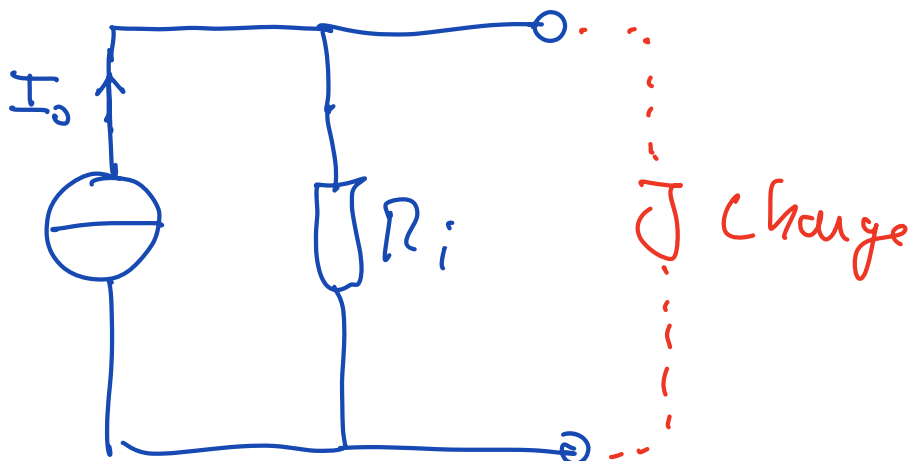
Courant I_{cc} :

$$U = 0$$

$$0 = U_0 - R_i \cdot I_{cc}$$

$$I_{cc} = \frac{U_0}{R_i}$$

4.2.6 Source de courant réelle:



4.3 Elément de base:

Résistance



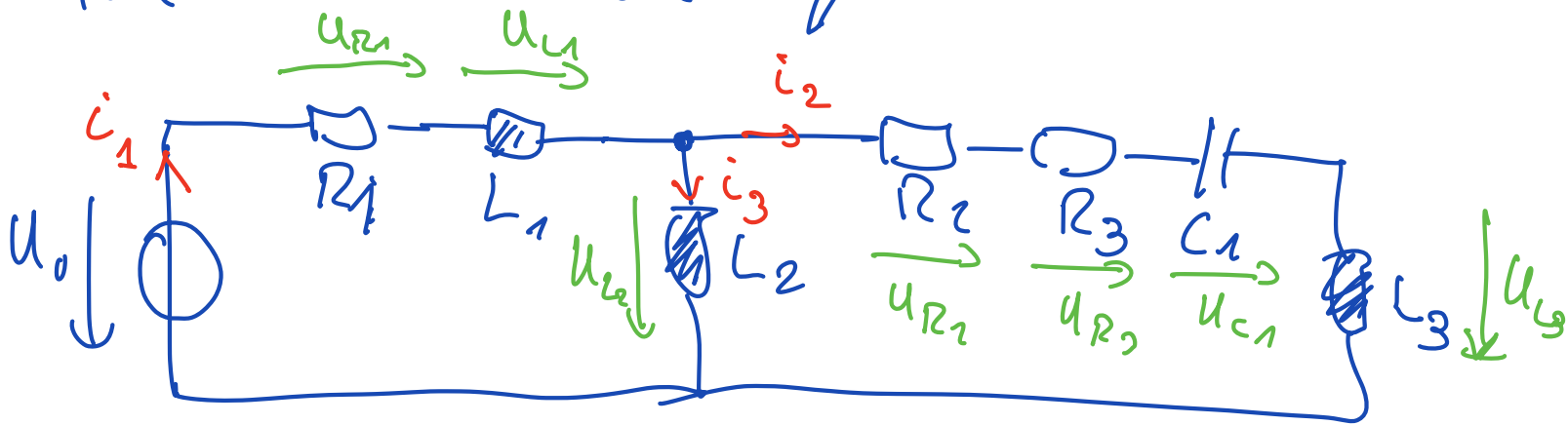
Inductance



Capacité

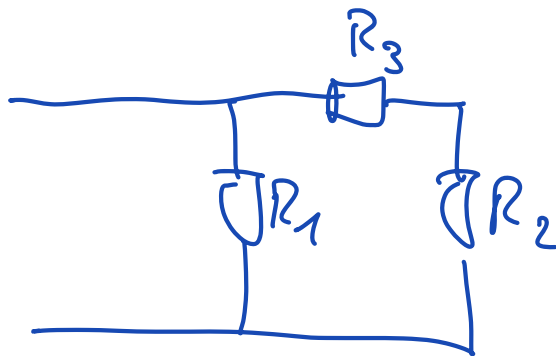


4.4 Schéma électrique :



Recap: Quiz :

4: // $\rightarrow$ même tension aux bornes



R_1 n'est pas en // avec R_2 !

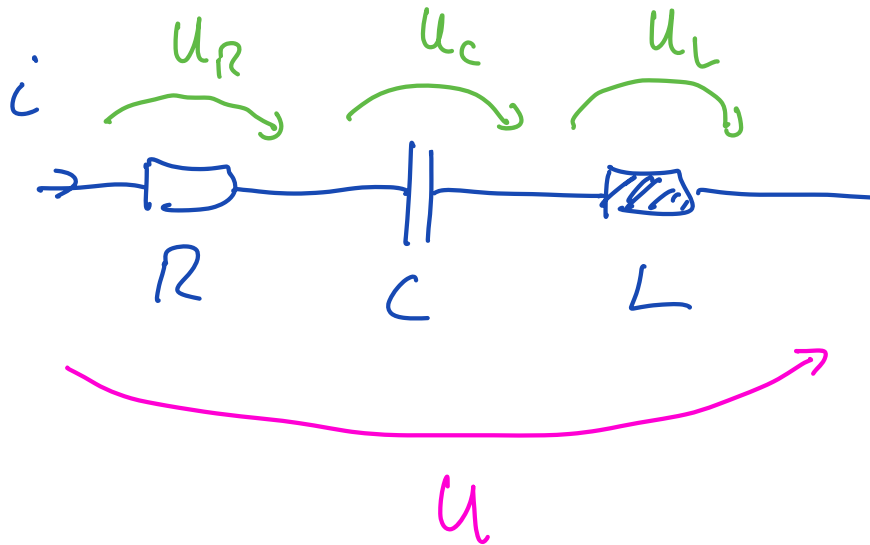
7: Source idéale seule impossible.

8: Source idéale est toujours constante

9: Impossible de mettre des sources de courant en série.

5. Combinaison simple d'éléments linéaires

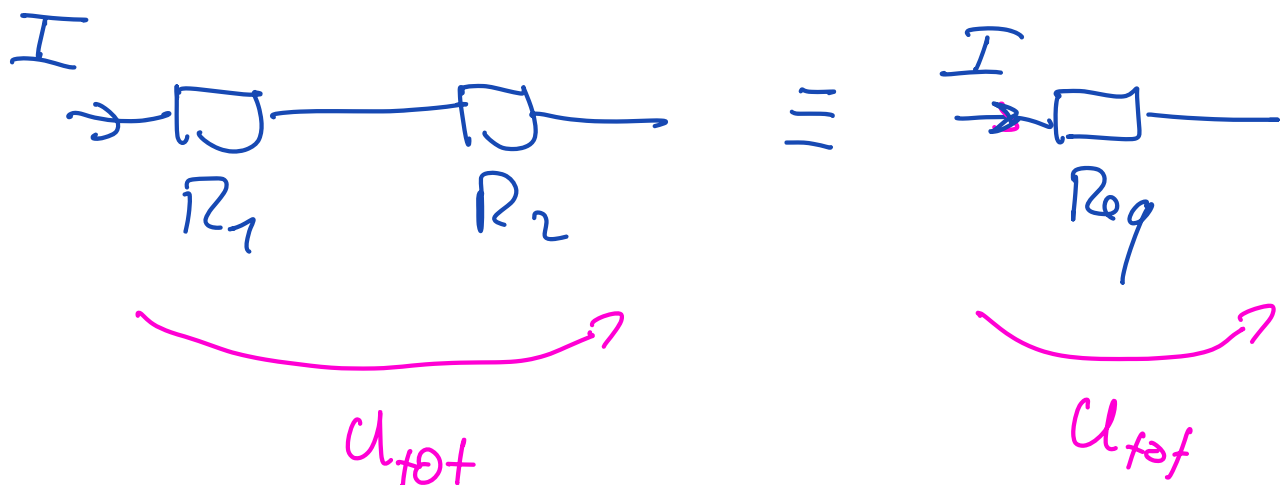
5.2 Mise en série :



Série : parcouru par le même courant

$$i_R = i_L = i_C \rightarrow \text{Série}$$

5.2.2 Mise en série de la résistance



$$U_{\text{tot}} = U_{R_1} + U_{R_2}$$

$$U_{\text{tot}} = R_{\text{eq}} \cdot I$$

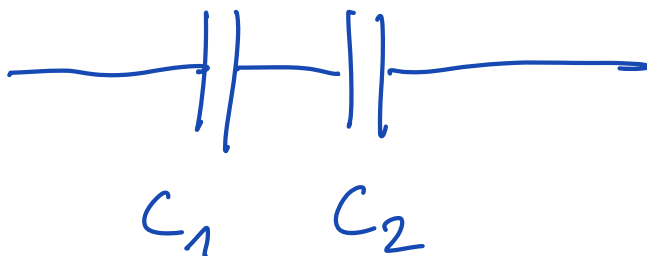
$$= R_1 I + R_2 I$$

$$= (R_1 + R_2) I = R_{\text{eq}} \cdot I$$

$$\Rightarrow R_{\text{eq}} = R_1 + R_2$$

En série $R_{\text{eq}} = \sum_{K=1}^m R_K$ ($m = \text{nb de } R$)

5.2.3 Mise en Série des C



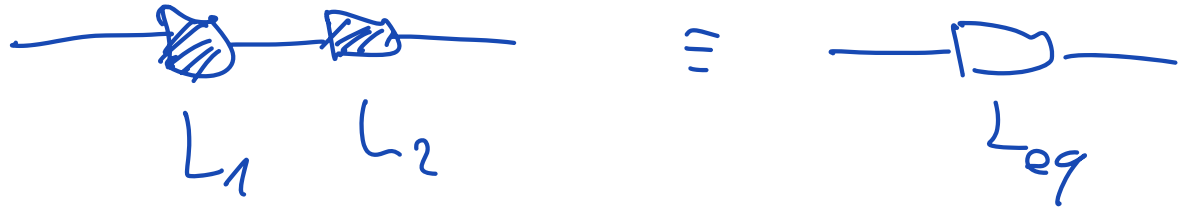
$$\equiv C_{\text{eq}} = ?$$



Série $C_{\text{eq}} = \frac{1}{\sum_{K=1}^m \frac{1}{C_K}}$

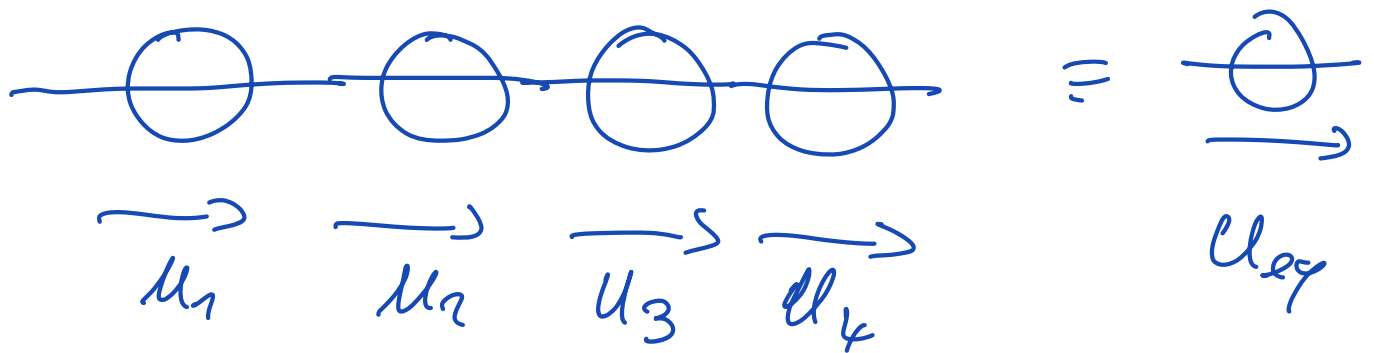
$m = \text{nb de } C$

5.2.6 Mise en série des L

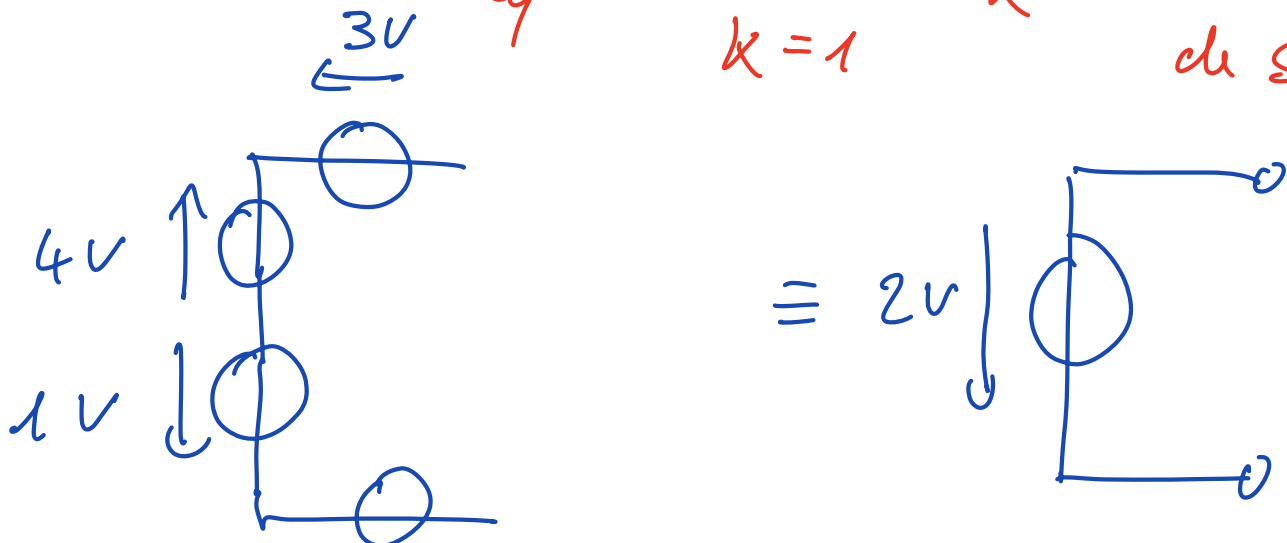


Série $L_{eq} = \sum_{k=1}^m L_k$ $m = nb \text{ de } L$

5.2.7 Mise en série de Source de tension

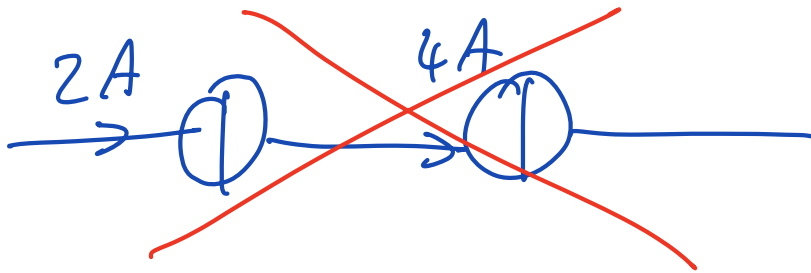


Série $U_{eq} = \sum_{k=1}^m U_k$ $m = nb \text{ de source}$



$\xrightarrow{2V}$

5.2.9 Mise en série de source de courant

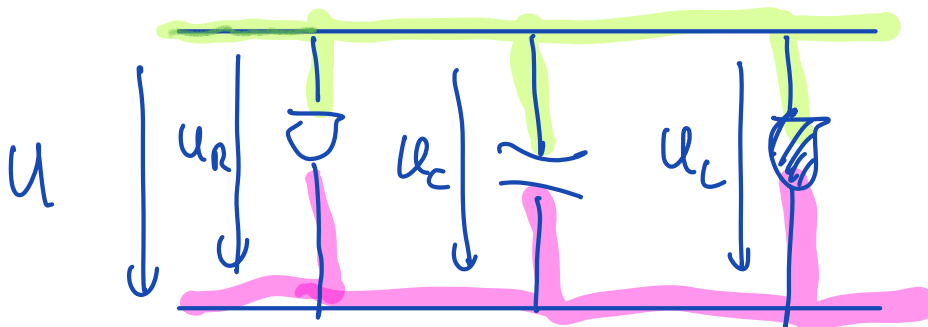


impossible

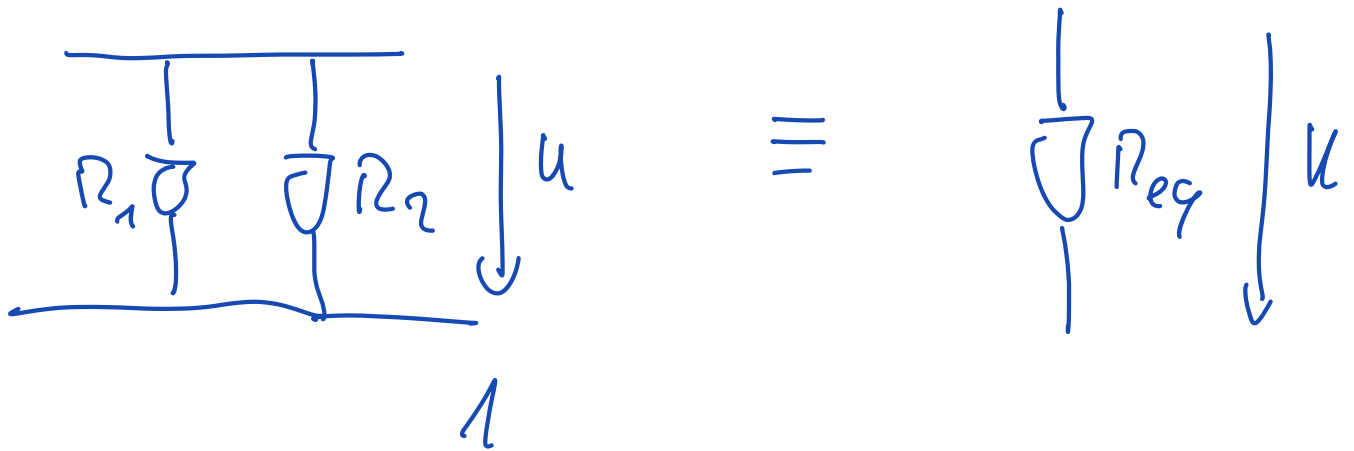
$\Rightarrow$ Impossible sauf si toutes les sources ont le même courant

5.3.2 Mise en // des R :

Définition : Toutes les bornes des éléments sont au même potentiel



$$U_R = U_C = U_L$$

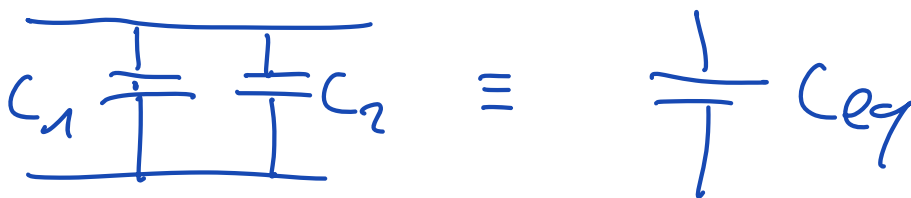


$$R_{eq} = \frac{1}{\sum_{k=1}^m \frac{1}{R_k}}$$

$$m = \text{nb de } R$$

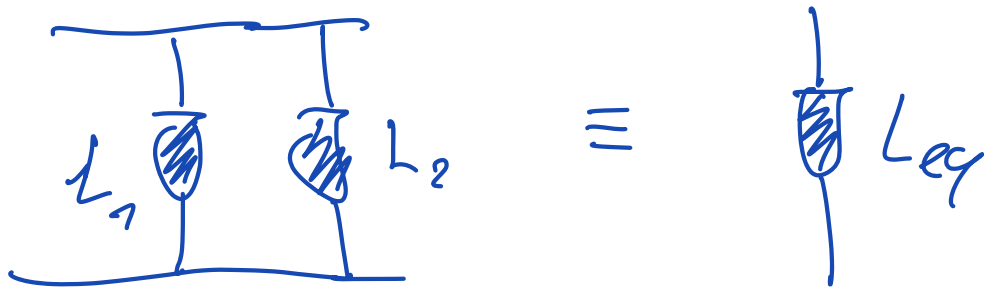
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

5.3.5 Mise en // des \$C\$:



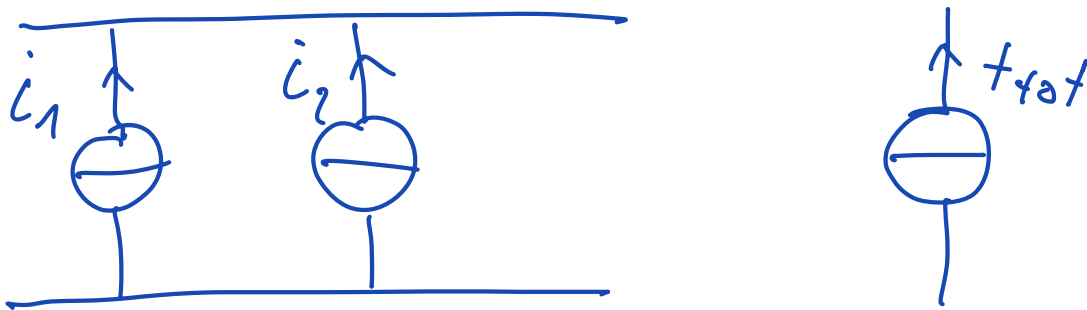
$$C_{eq} = \sum_{k=1}^m C_k \quad m = \text{nb de } C$$

5.3.6 Mise en // des L



$$// L_{eq} = \frac{1}{\sum_{k=1}^m \frac{1}{L_k}} \quad m = \text{nb de } L$$

5.3.7 Mise en // des sources de courant

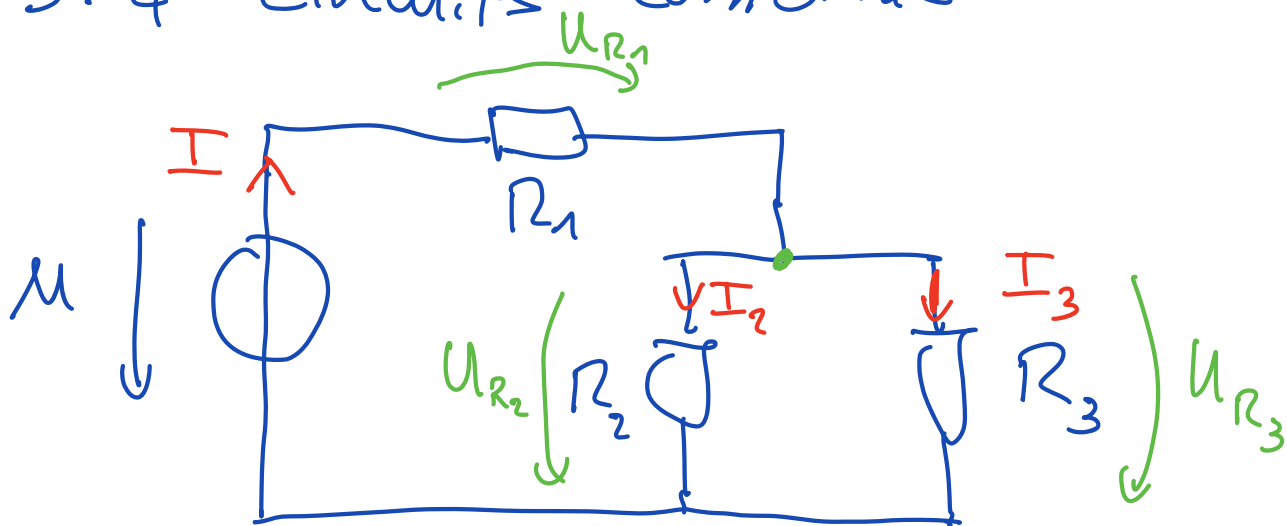


$$i_{tot} = \sum_{k=1}^m i_k$$

Mise en // de sources de tension

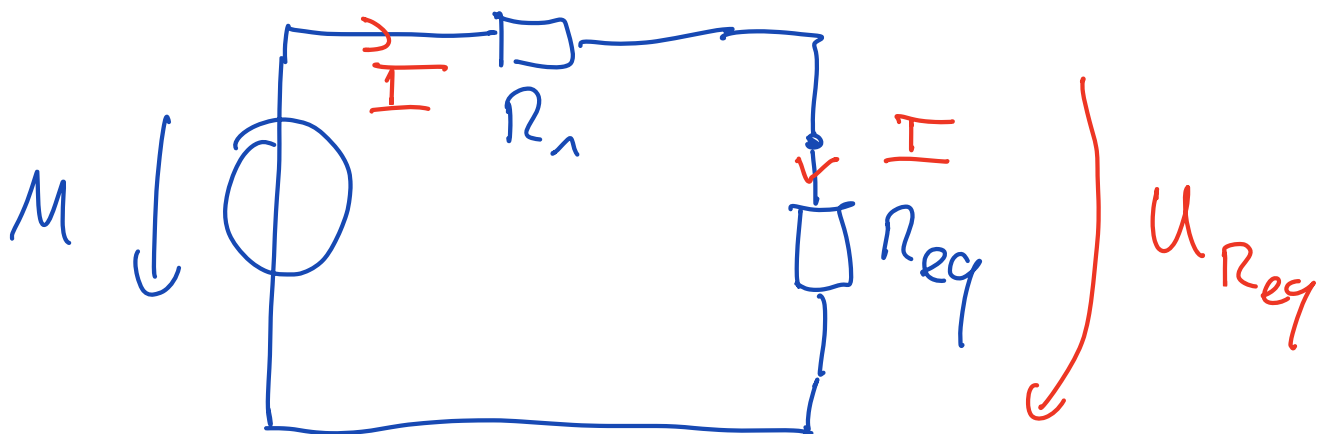
est impossible sauf si toutes les tensions ont la même valeur

5.4 Circuits combinés :

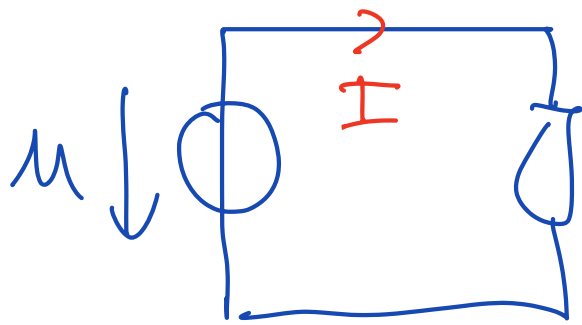


$$I = ? \quad I_2 = ? \quad I_3 = ?$$

$$R_2 \parallel R_3$$



$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3}$$



$$R_{tot} = R_1 + R_{eq}$$

$$= R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

$$U = R_{tot} \cdot I$$

$$I = \frac{U}{R_{tot}}$$

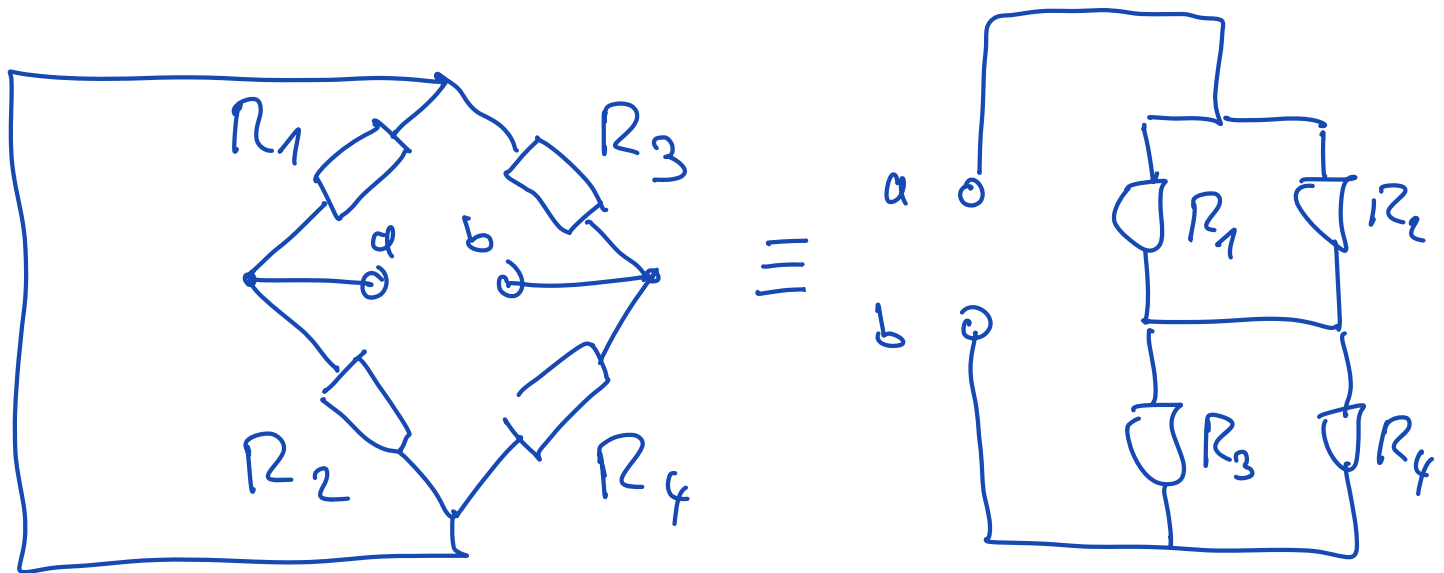
$$U_{R_2} = U_{R_3} = U_{R_{eq}} = R_{eq} \cdot I$$

$$= R_{eq} \cdot \frac{U}{R_{tot}}$$

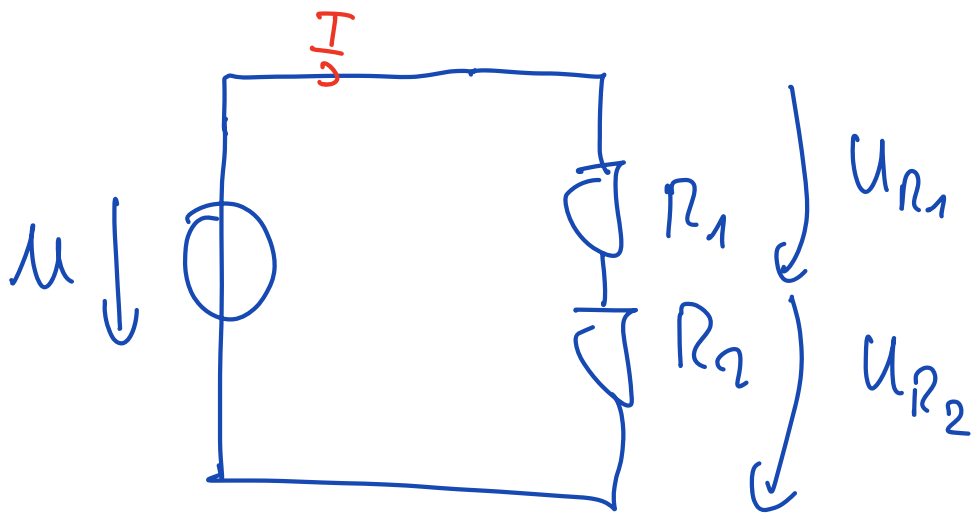
$$I_2 = \frac{U_{R_2}}{R_2} = \frac{U_{R_{eq}}}{R_2}$$

$$I_3 = \frac{U_{R_3}}{R_3} = \frac{U_{R_{eq}}}{R_3}$$

5.4.3 Exemple :



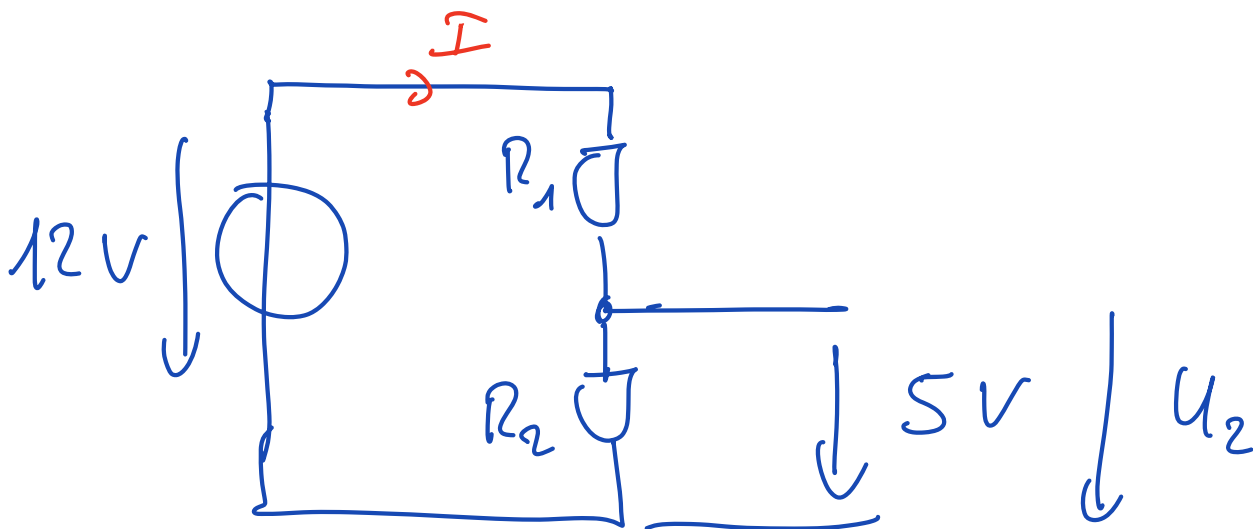
5.5.1 Diviseur de tension :



$$U = U_{R_1} + U_{R_2}$$
$$= (R_1 + R_2) I$$

$$I = \frac{U}{R_1 + R_2}$$

$$\underline{U_{R_2} = R_2 \cdot I = \frac{R_2}{R_1 + R_2} \cdot U}$$



$$U_2 = \frac{R_2}{R_1 + R_2} \cdot U$$

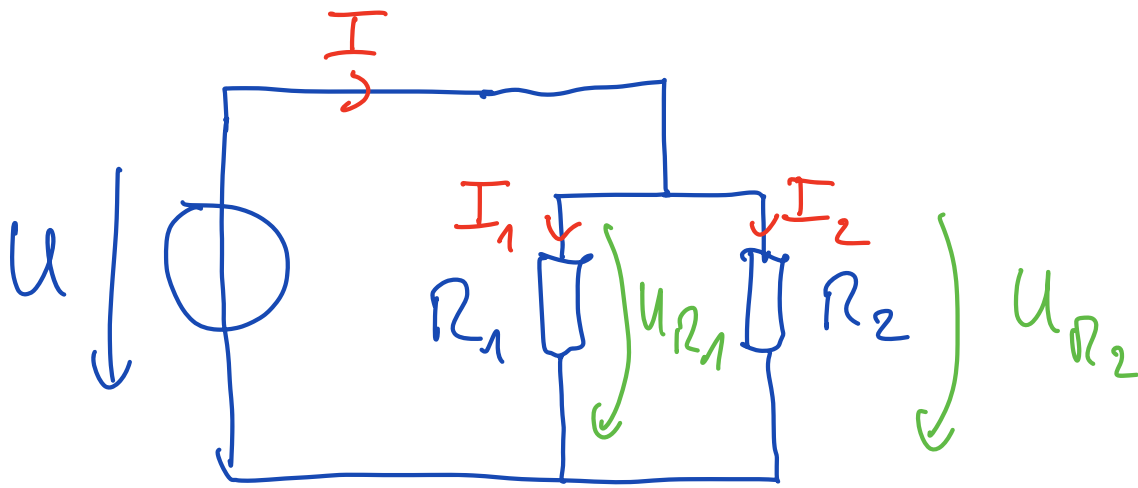
$$5 = \frac{R_2}{R_1 + R_2} \cdot 12$$

$$5R_1 = 7R_2$$

$$R_1 = 100 \text{ k}\Omega$$

$$R_2 = 71,5 \text{ k}\Omega$$

5.5.4 Diviseur de courant :



$$U = U_{R1} = U_{R2}$$

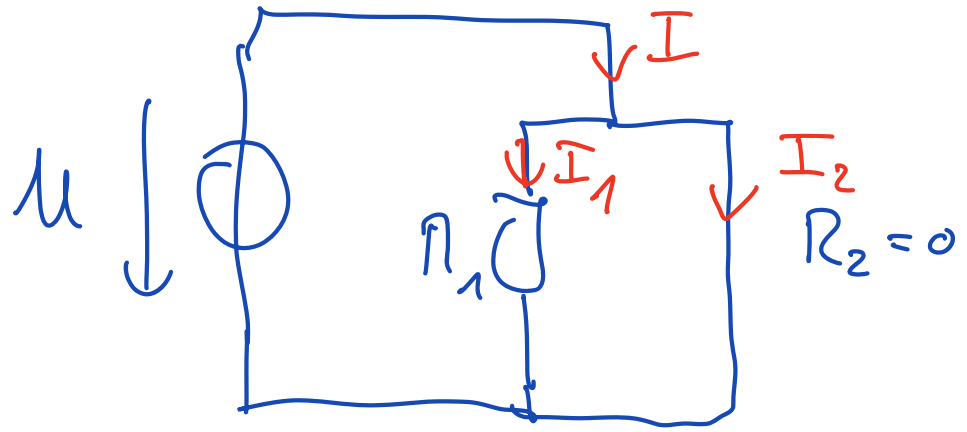
$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \quad I = \frac{U}{R_{eq}}$$

$$U_{R2} = R_2 \cdot I_2 = U = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot I$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I$$

$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

$S_i :$



$$I_2 = \frac{R_1}{R_1 + \cancel{R_2}} \cdot I = I$$

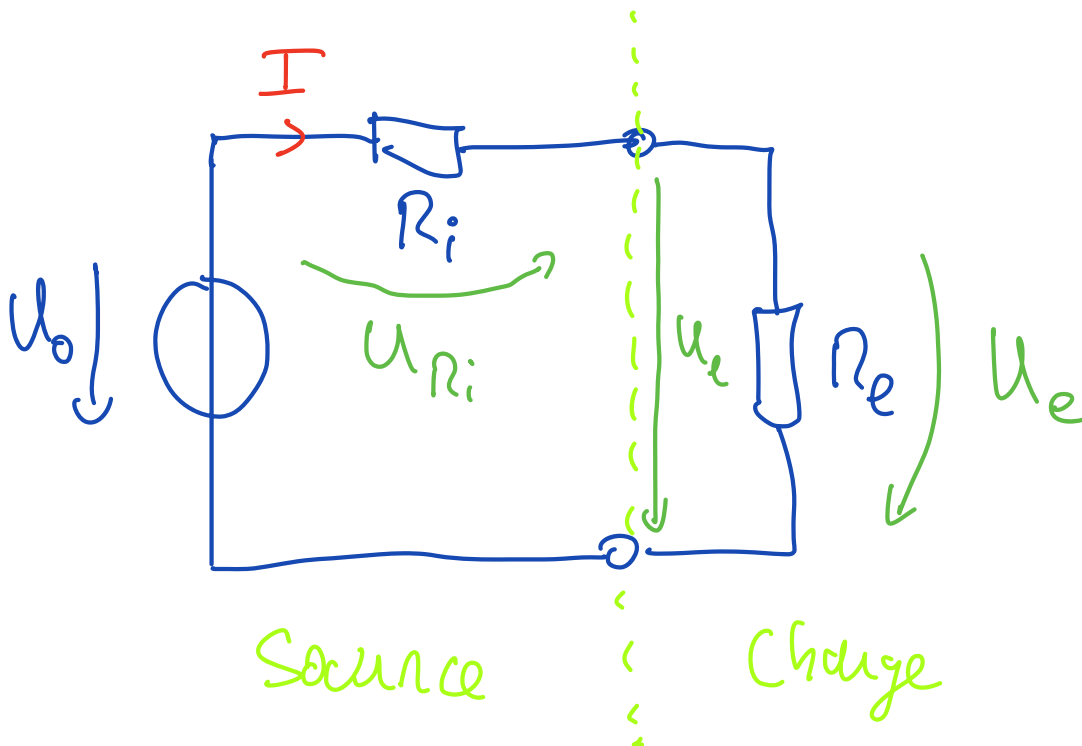
$$I_1 = 0$$

5.6 Méthodes de résolution:

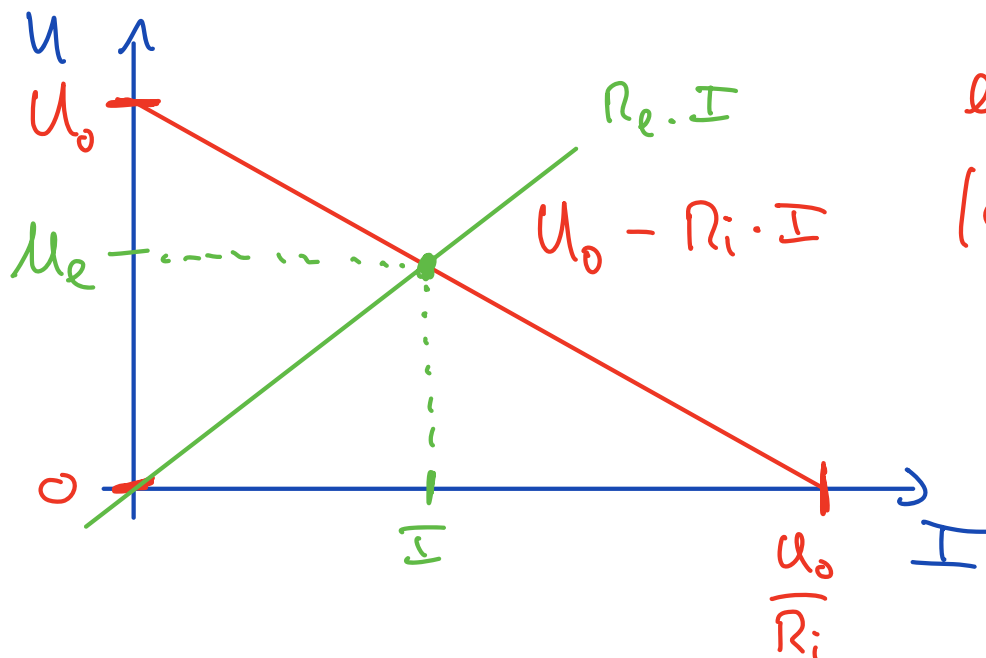
- Redessiner le schéma
- Définir toutes les grandeurs
 $U, I, \dots \rightarrow$ indice
- Définir le sens des flèches
- Réduire le schéma, série ou //

• Analyse !

5.6.2 Source de tension réelle :



$$U_e = U_0 - U_{R_i} = U_0 - R_i I$$



en court-circuit

$$(cc) : U_e = 0$$

$$I_{cc} = \frac{U_0}{R_i}$$

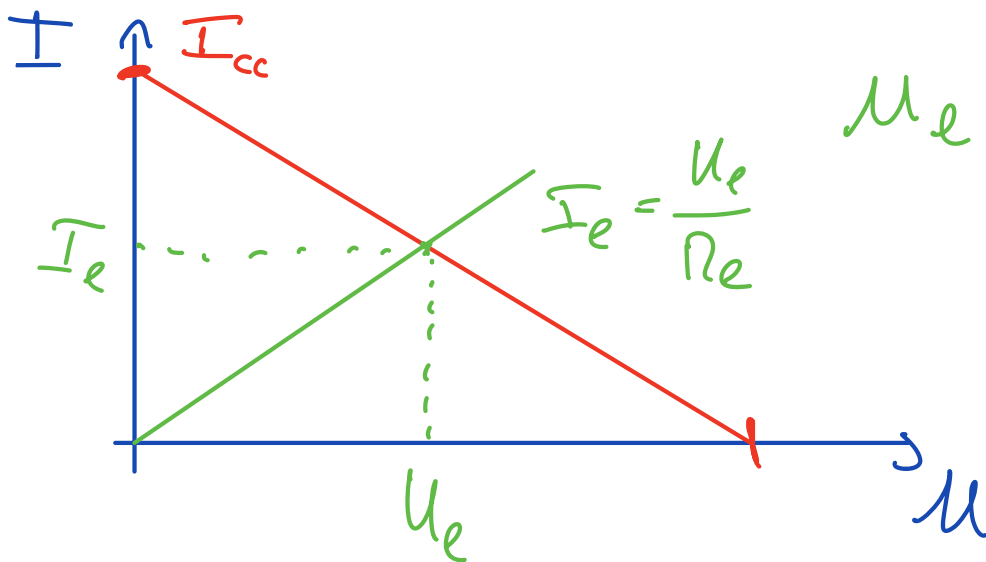
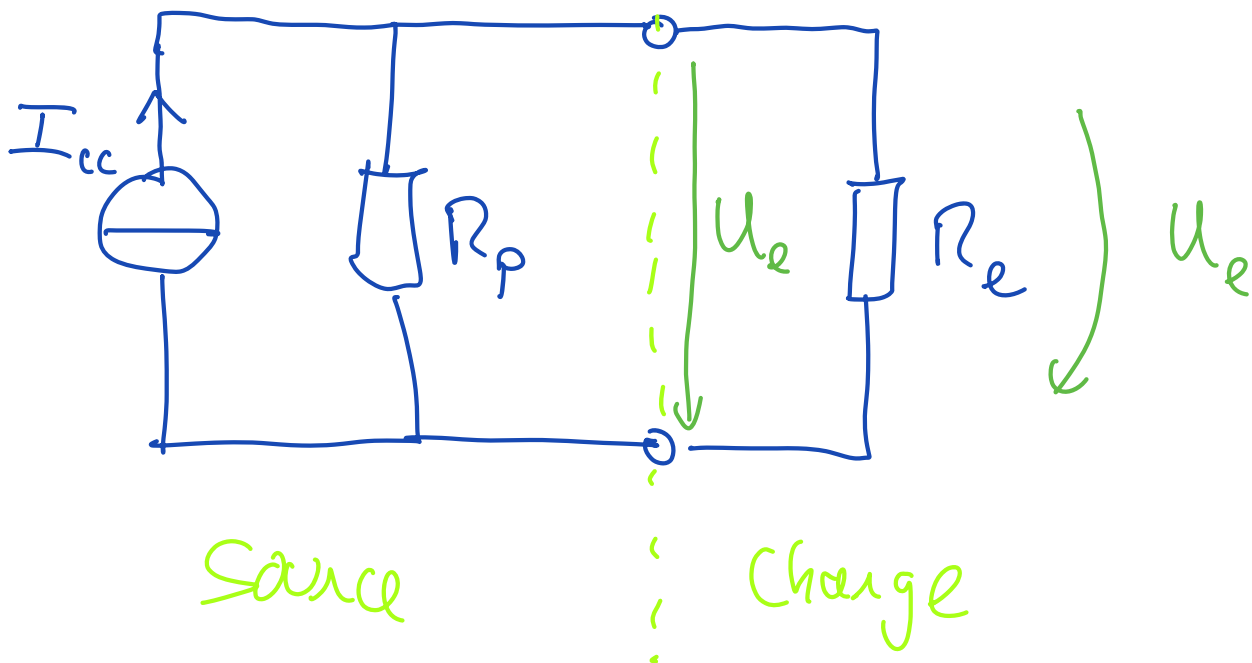
eq de la charge

$$U_e = R_e \cdot I$$

$$U_e = U_0 \cdot \frac{R_e}{R_e + R_i}$$

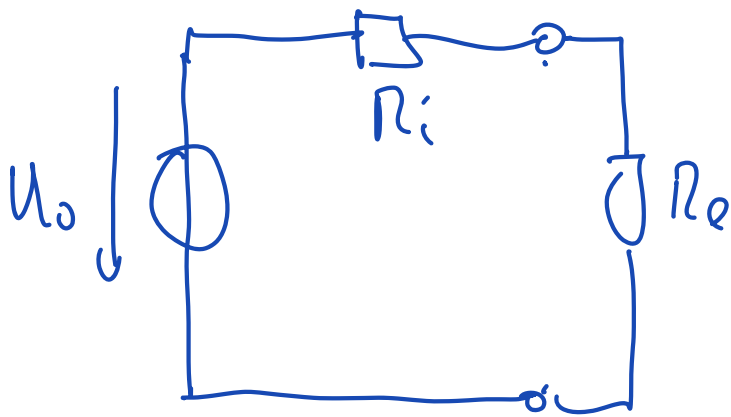
$$I_e = \frac{U_0}{R_i + R_e}$$

Source de courant réelle :



$$U_e = R_e \cdot I_e$$

5.6.3 Equivalence des Sources de tension et courant réelles



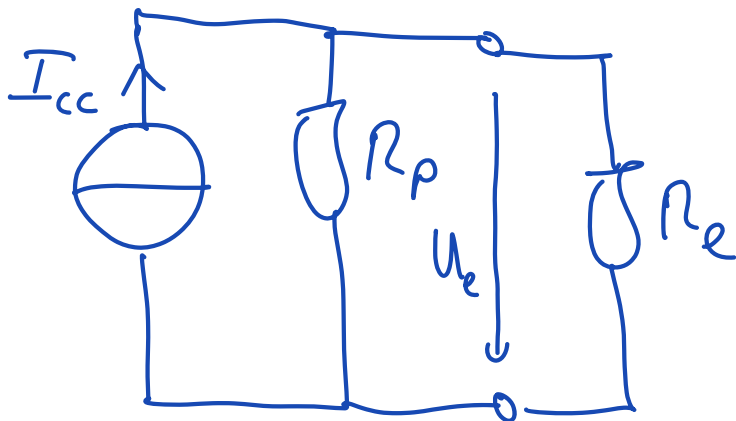
court-circuit
 $R_e = 0$

$$U_{e_{cc}} = 0$$

$$I_{e_{cc}} = \frac{U_0}{R_i}$$

circuit ouvert
 $R_e \rightarrow \infty$

$$U_{e_0} = U_0$$



$$I_{e_{cc}} = I_{cc}$$

$$U_{e_{cc}} = 0$$

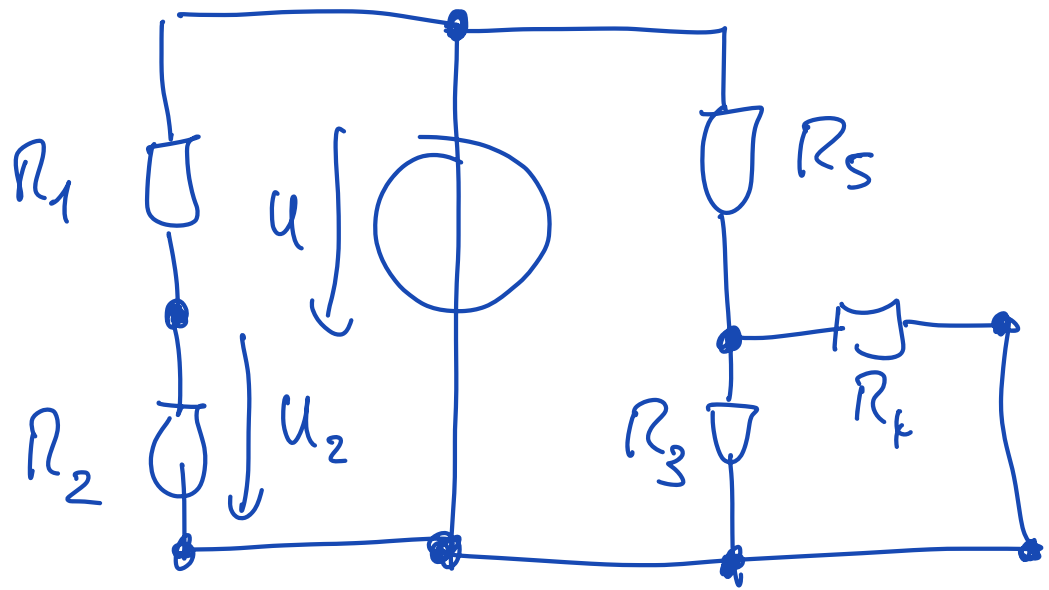
$$U_{e_0} = R_p \cdot I_{cc}$$

On pose : $U_{e_0} = R_p \cdot I_{cc} = U_0$

$$I_{e_{cc}} = \frac{U_0}{R_i} = I_{cc}$$

$$R_p = \frac{U_0}{I_{cc}} = \frac{U_0}{U_0/R_i} = R_i$$

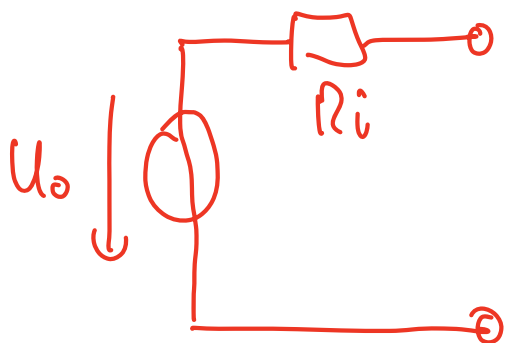
Quiz :



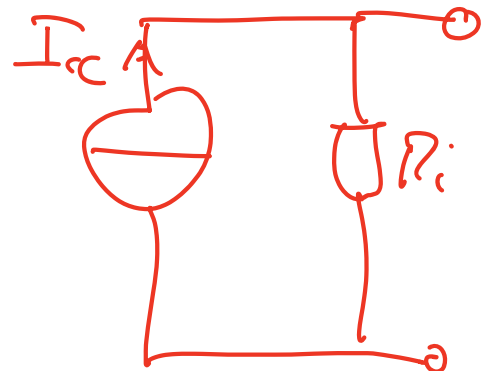
$$U = 12 \text{ V}$$

$$U_2 = U \cdot \frac{R_2}{R_1 + R_2} = 5 \text{ V}$$

En résumé :

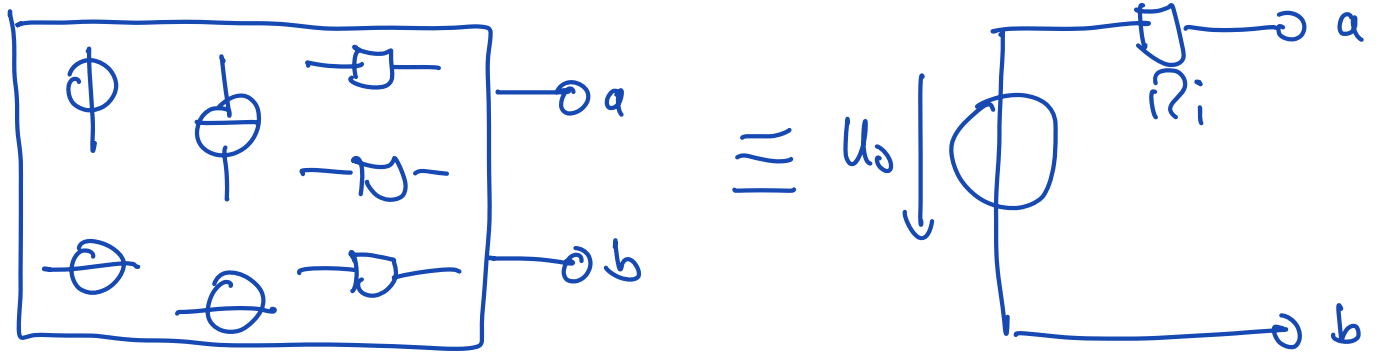


$\equiv$



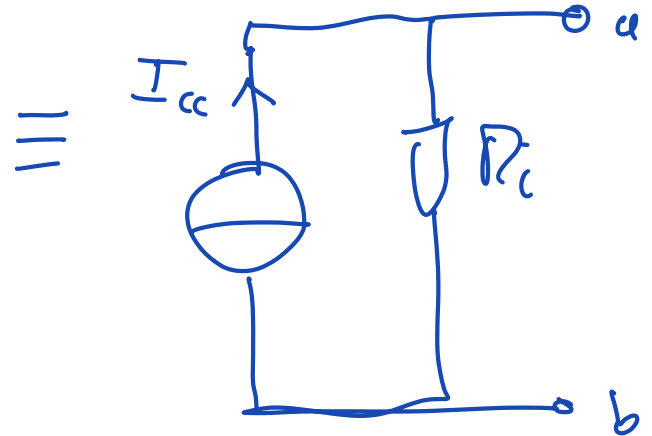
$$I_{cc} = \frac{U_0}{R_i}$$

5.7 Théorèmes de Thévenin et Norton:



U_0 = Tension à vide

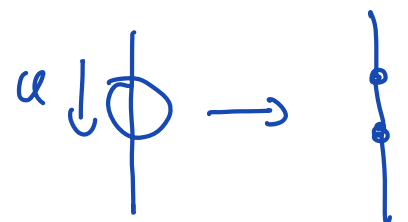
$$U_0 = U_{ab} \left| \begin{array}{l} \text{(à vide)} \\ I_{ab} = 0 \end{array} \right.$$

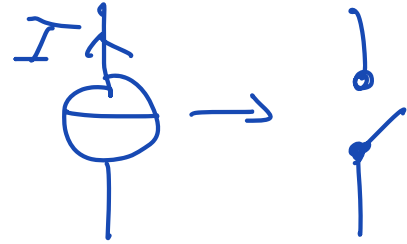


$$I_{cc} = I_{ab} \left| \begin{array}{l} \text{(en court-circuit)} \\ U_{ab} = 0 \end{array} \right.$$

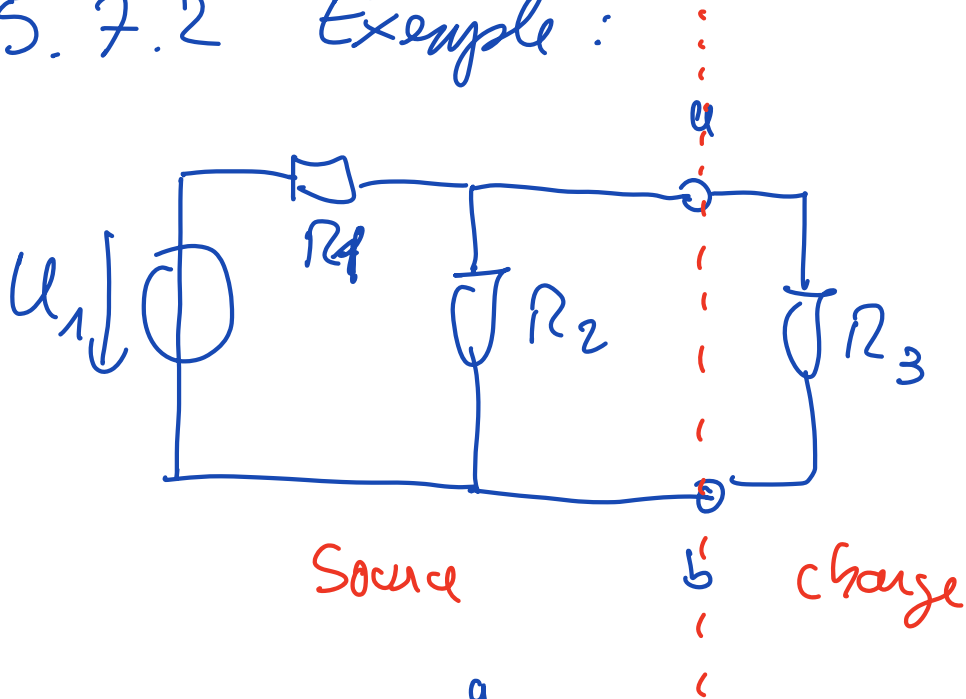
$$R_i = \frac{U_0}{I_{cc}} = R_{ab} \left| \begin{array}{l} U_j = 0 \\ I_j = 0 \end{array} \right.$$

Annuler une source $\Rightarrow$

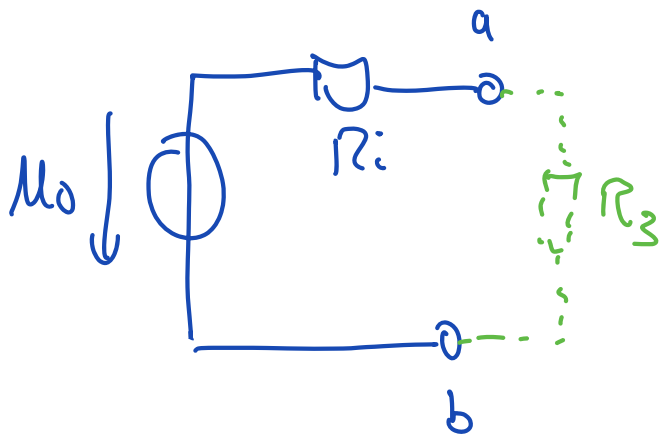




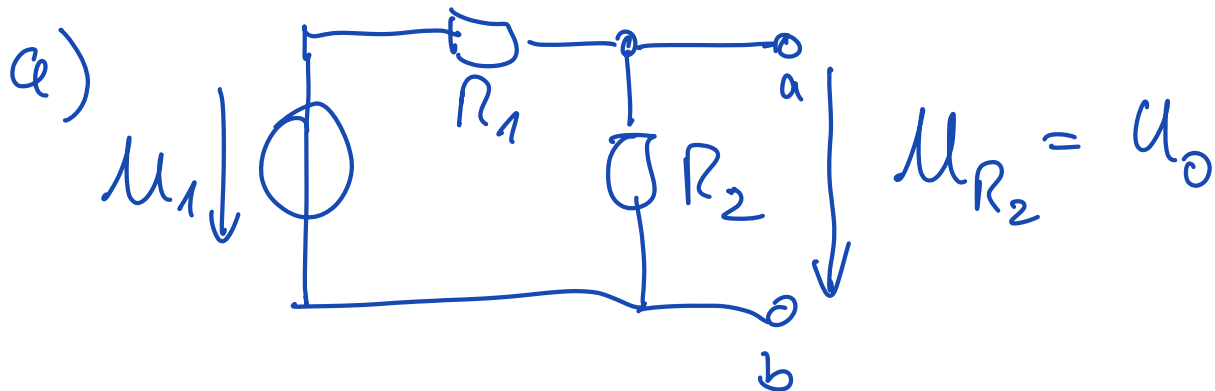
5.7.2 Example :



$$U_3 = f(R_3)$$

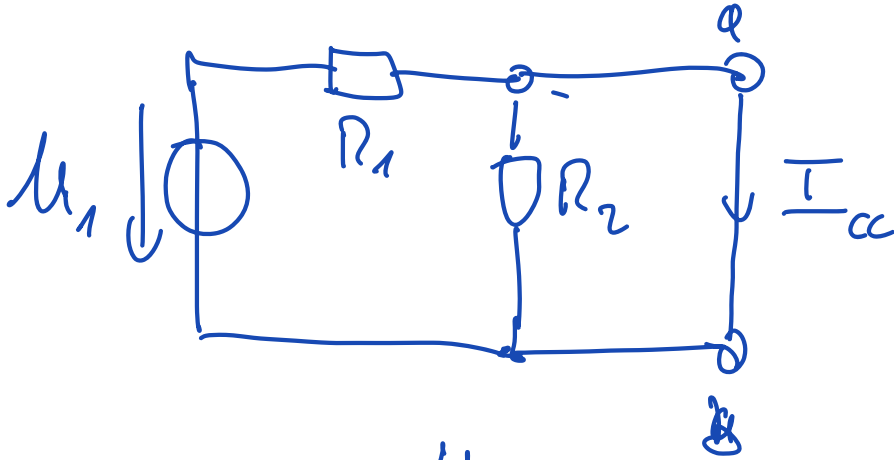


U_0 ? Tension à vide entre a et b ?



$$U_{R_2} = U_0 = U_1 \cdot \frac{R_2}{R_1 + R_2}$$

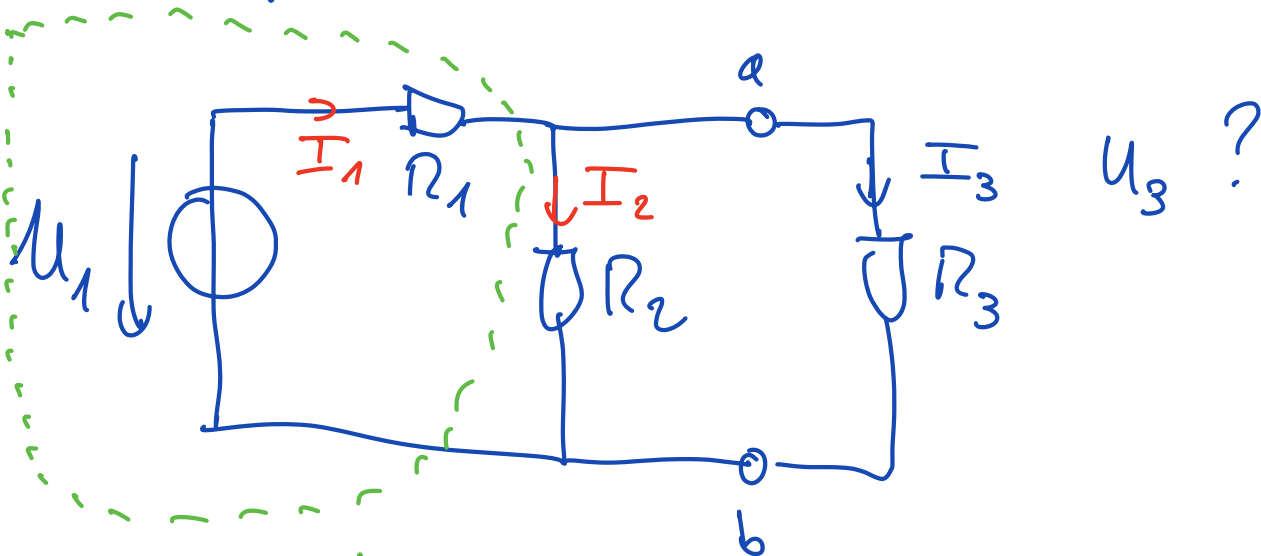
b) I_{cc} = courant de court circuit



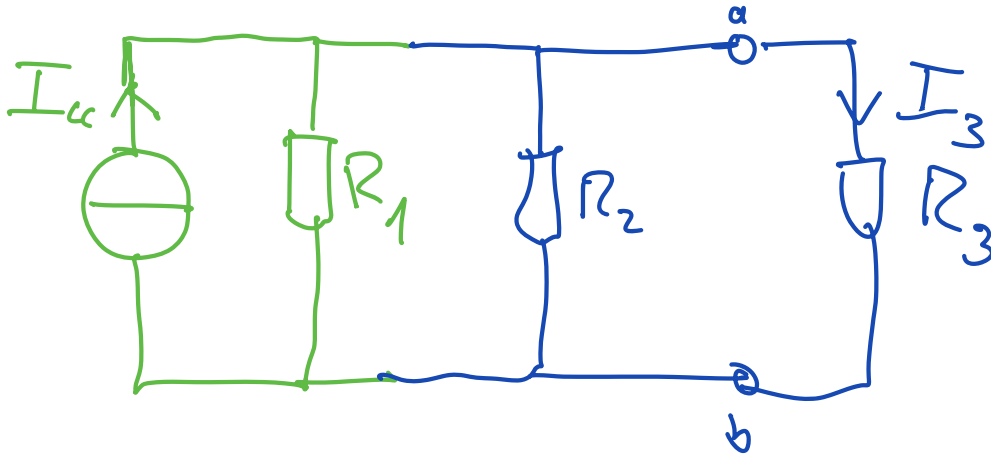
$$I_{cc} = \frac{U_1}{R_1}$$

c) $R_i = \frac{U_o}{I_{cc}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$

Autre possibilité :

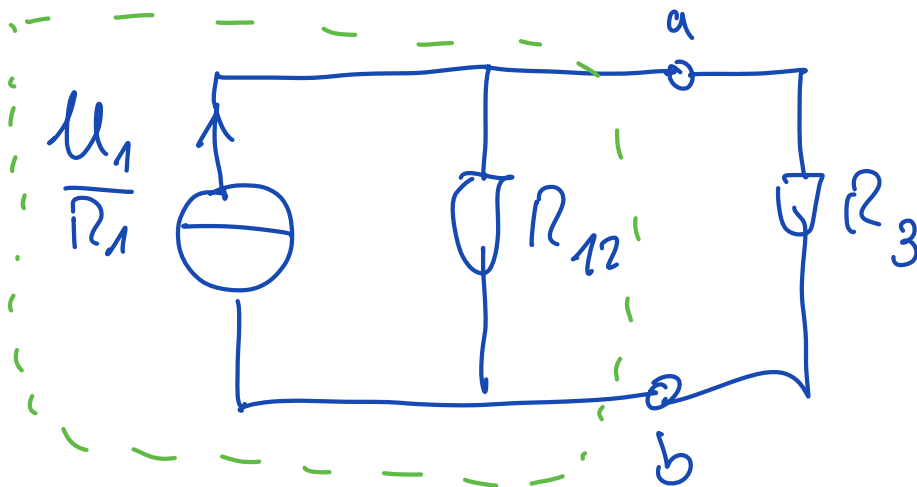


Source de tension
réelle

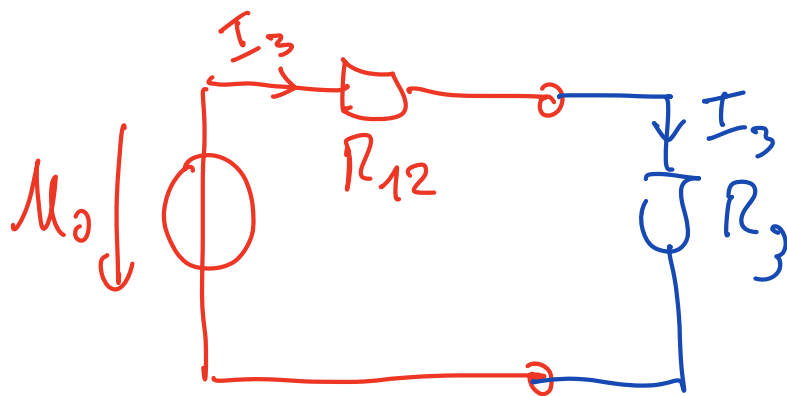


$$I_{cc} = \frac{U_1}{R_1}$$

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



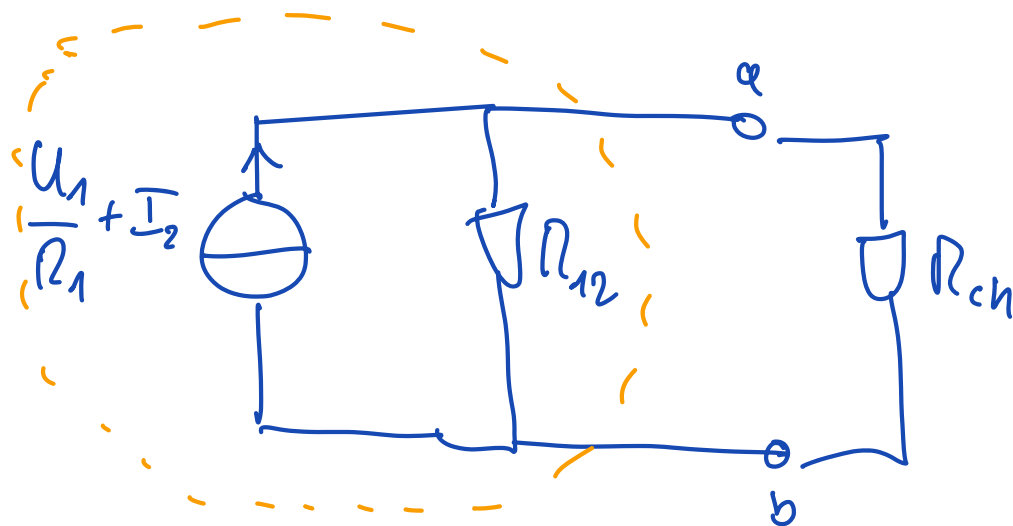
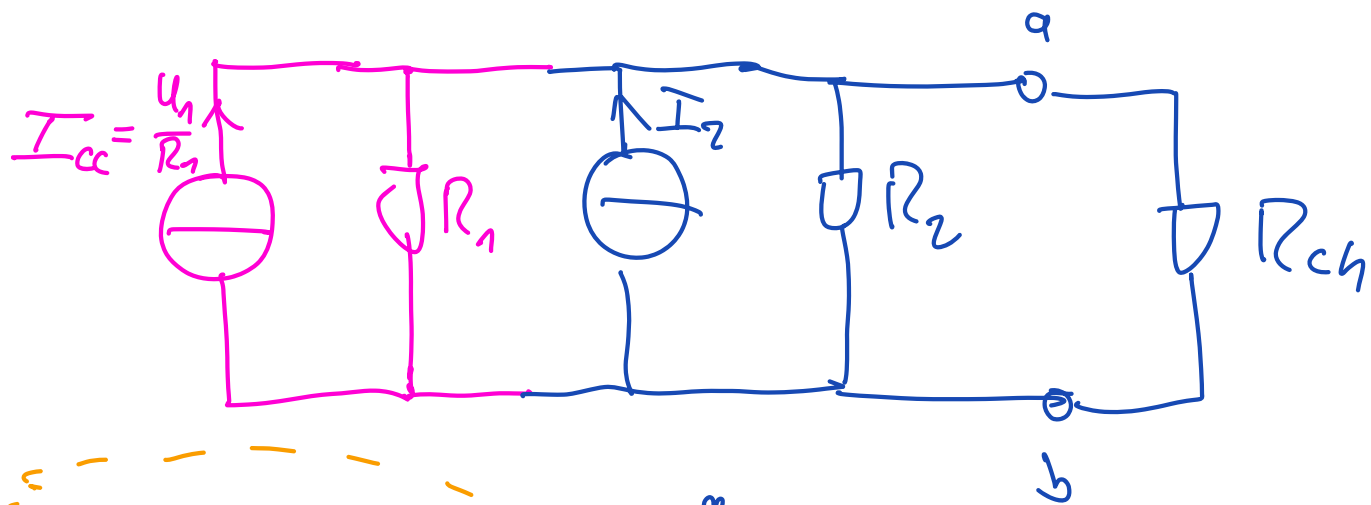
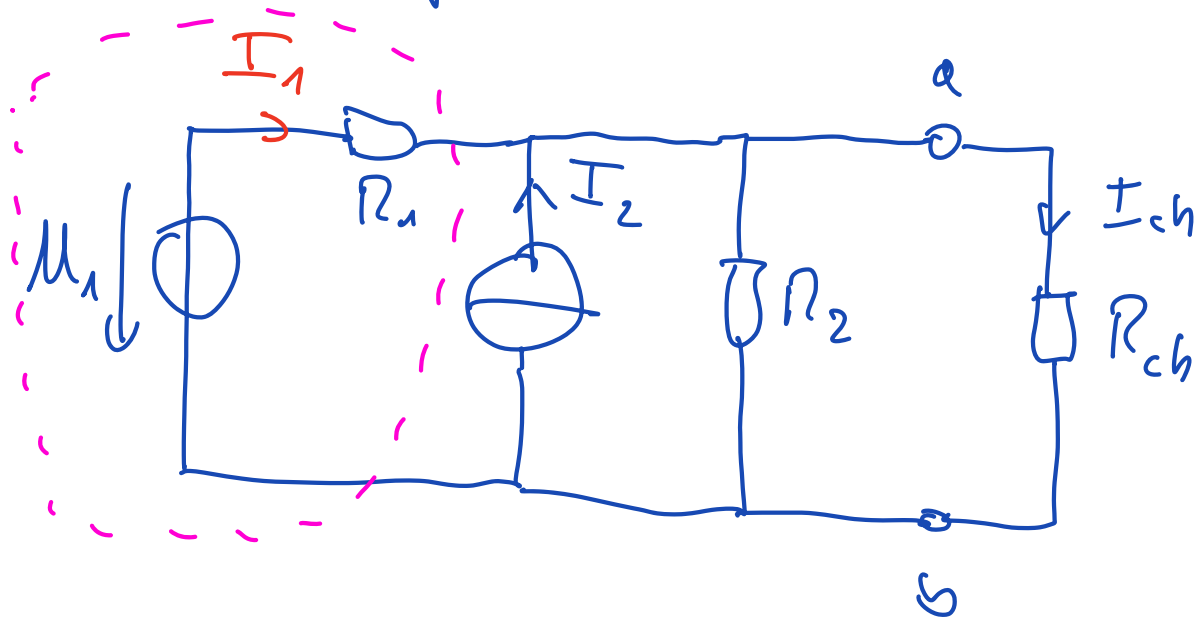
Source de courant réel



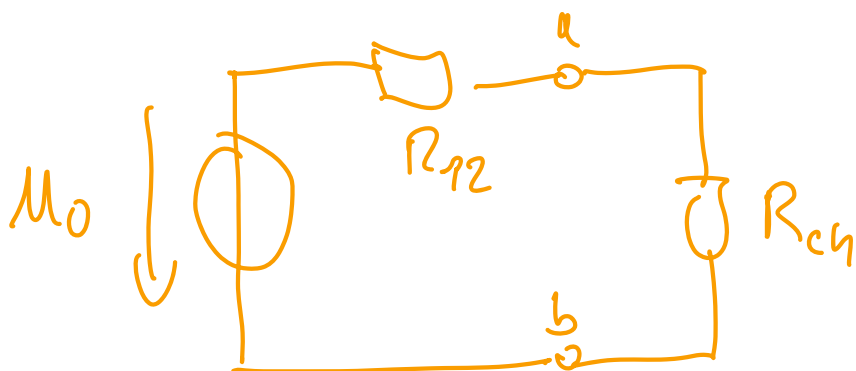
$$U_0 = R_{12} \cdot \frac{U_1}{R_1}$$

$$I_3 = \frac{U_0}{R_{12} + R_3}$$

Another example:



$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

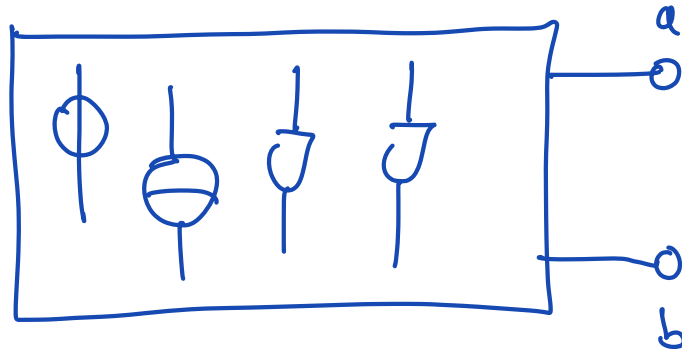


$$U_0 = R_{12} \left(\frac{U_1}{R_1} + I_2 \right)$$

$$I_{ch} = \frac{U_0}{R_{12} + R_{ch}}$$

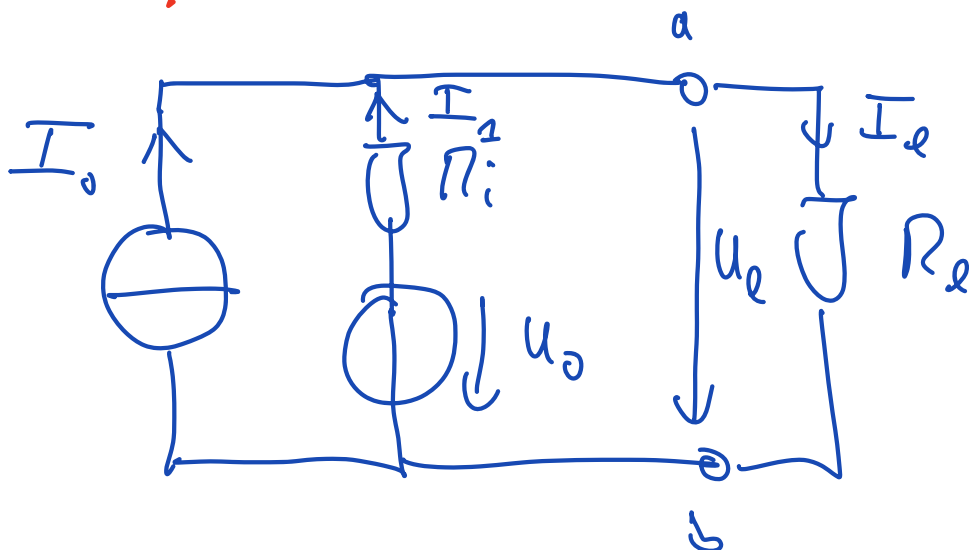
5.8 Principe de Superposition

Définition :



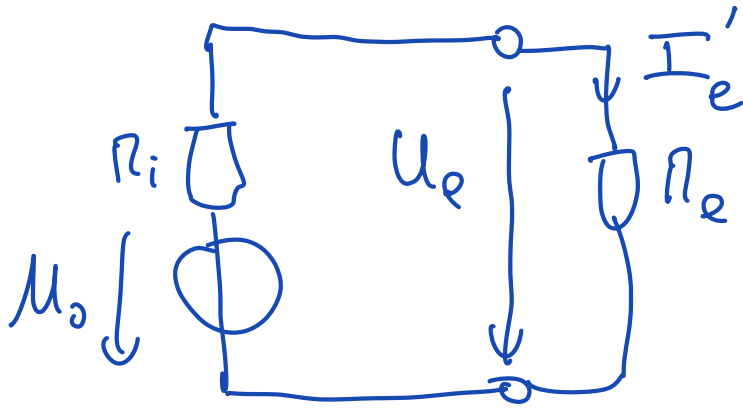
L'action résultante est la somme algébrique des actions séparées de chaque source, les autres étant annulées

Le système doit être linéaire !!



I_e, u_e ?

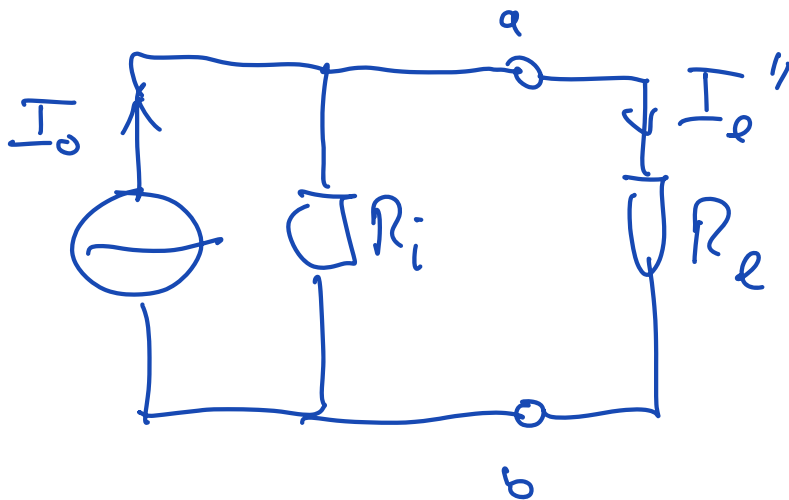
1) On annule la source de courant :



$$I_e' = \frac{U_0}{R_i + R_e}$$

$$U_e' = U_0 \cdot \frac{R_e}{R_e + R_i}$$

2) On annule la source de tension :



$$I_e'' = I_0 \cdot \frac{R_i}{R_i + R_e}$$

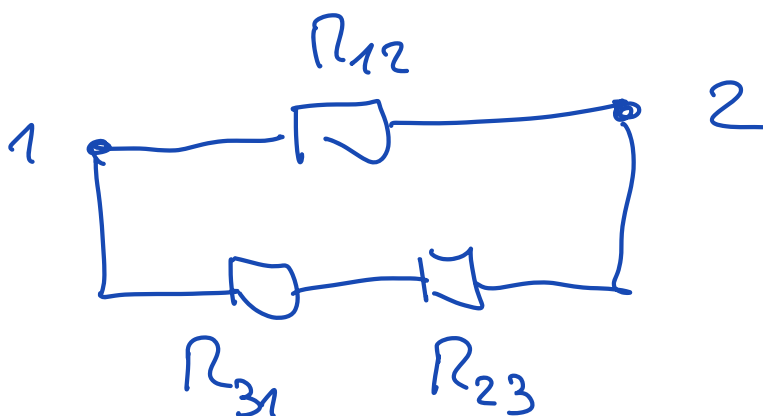
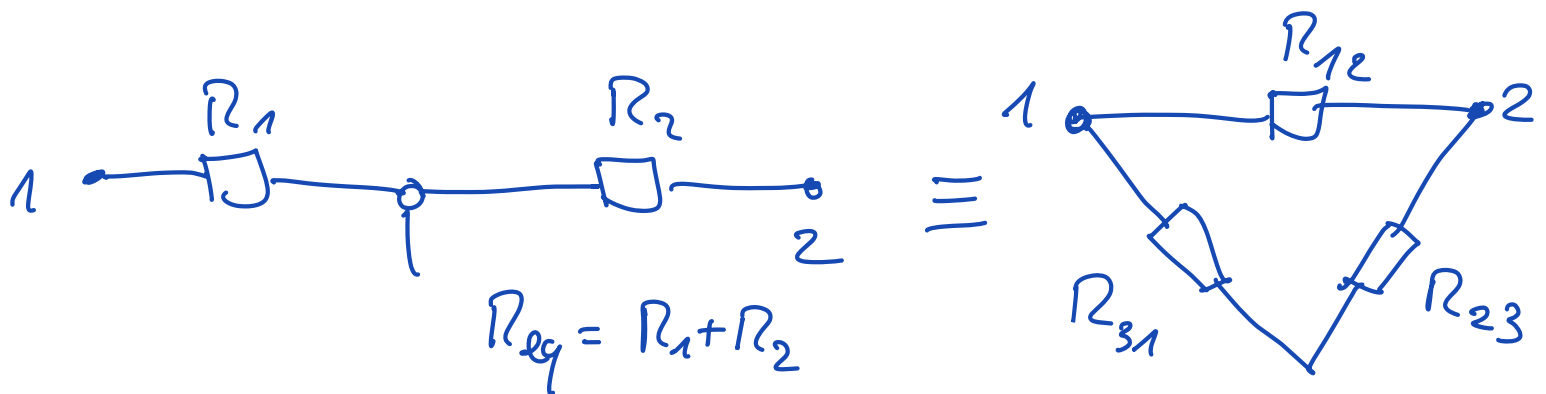
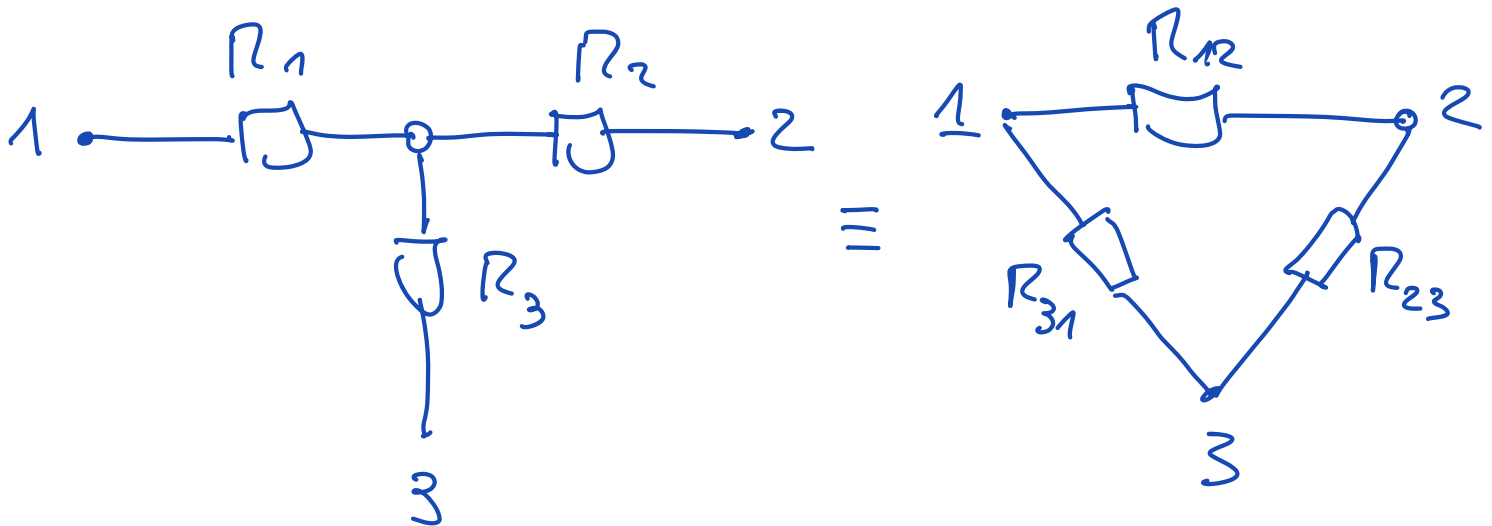
$$U_e'' = R_e \cdot I_e'' = I_0 \cdot \frac{R_e \cdot R_i}{R_e + R_i}$$

$$I_e = I_e' + I_e''$$

$$M_e = M_e' + M_e''$$

5.9 Transformation Étoile - Triangle

il s'agit d'un tripole :



$$R_{eq} = \frac{R_{12} \cdot (R_{31} + R_{23})}{R_{12} + R_{31} + R_{23}}$$

$$R_1 = \frac{R_{31} \cdot R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

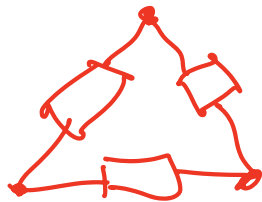
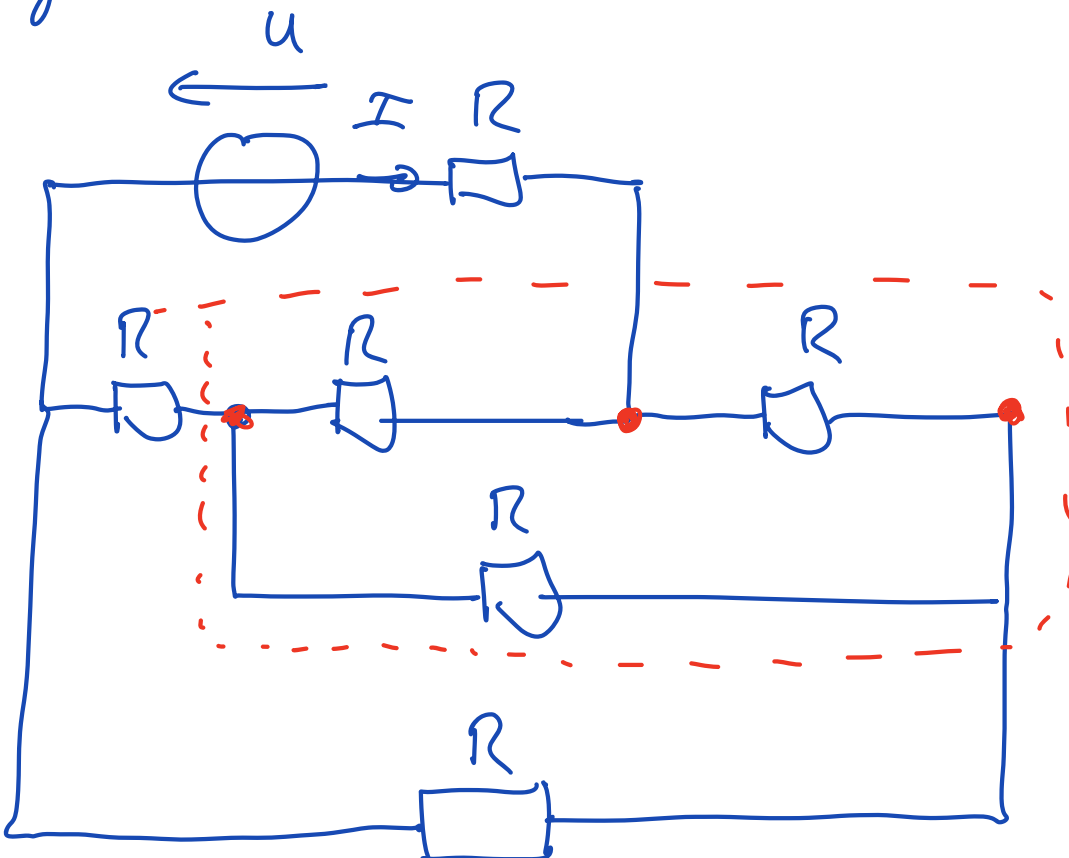
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

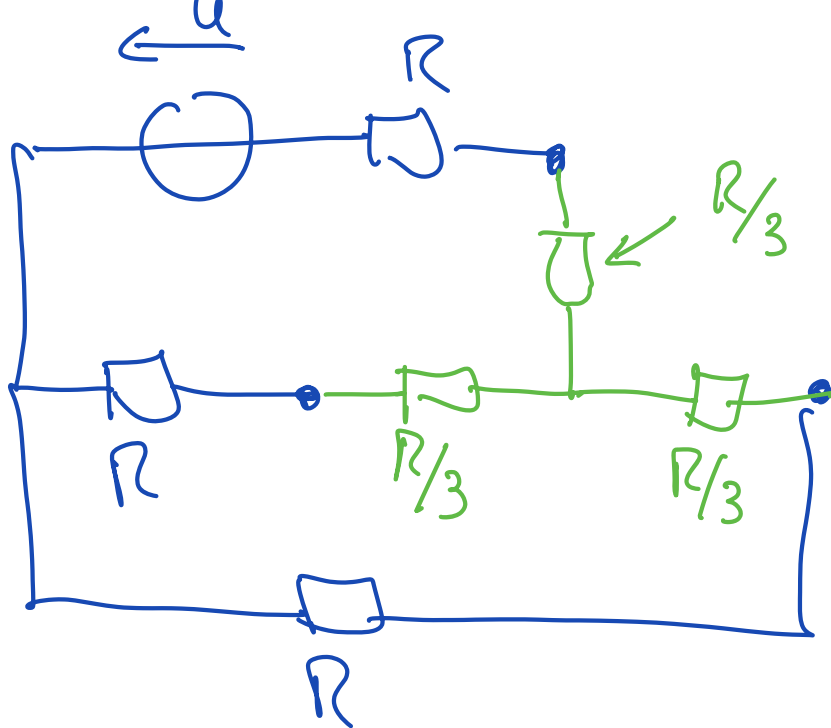
$$R_{23} = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

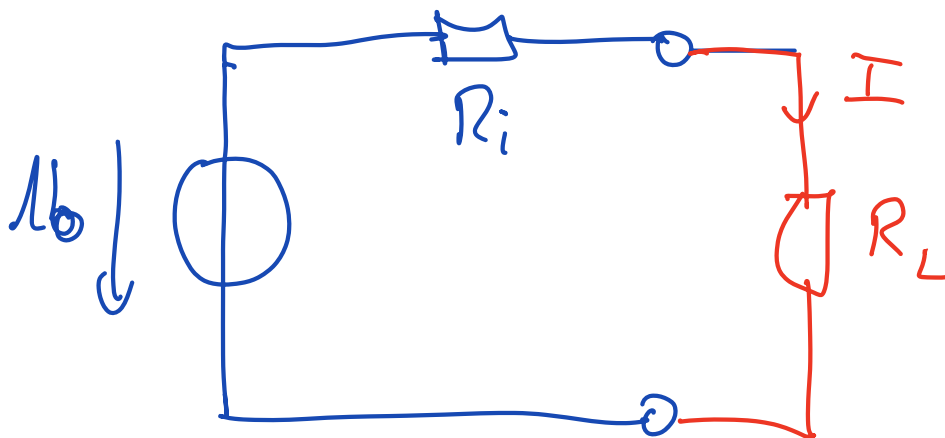
$$R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

Example :





5.11 Puissance Maximum transmise
par un dipôle :



Puissance
transmise à
 $R_L \rightarrow P_{\text{ox}}$

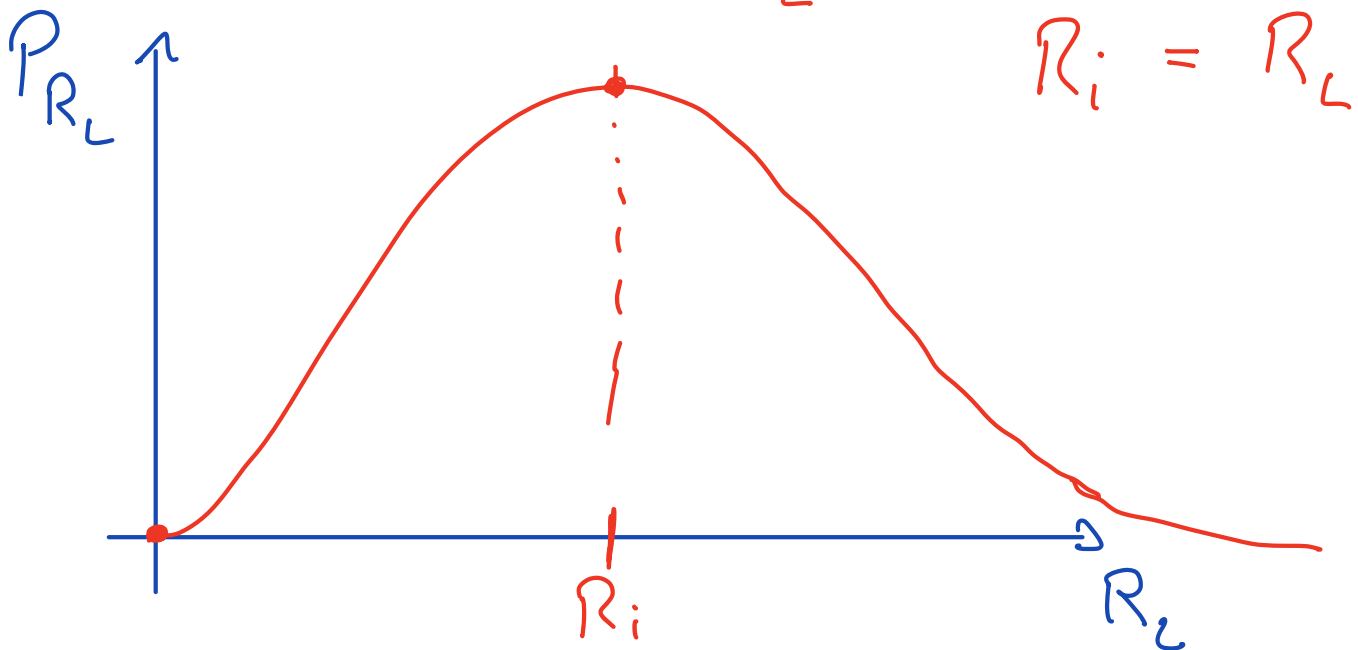
$$P_{R_L} = R_L \cdot I^2 = U_{R_L} \cdot I$$

$$I = \frac{U_0}{R_i + R_L}$$

$$P_{R_L} = R_L \frac{U_0^2}{(R_i + R_L)^2}$$

Qui doit valoir R_L par rapport à R_i pour P_{R_L} max

Max : $\rightarrow \frac{dP_{R_L}}{dR_L} = 0$



$\rightarrow$ d'adaptation de puissance

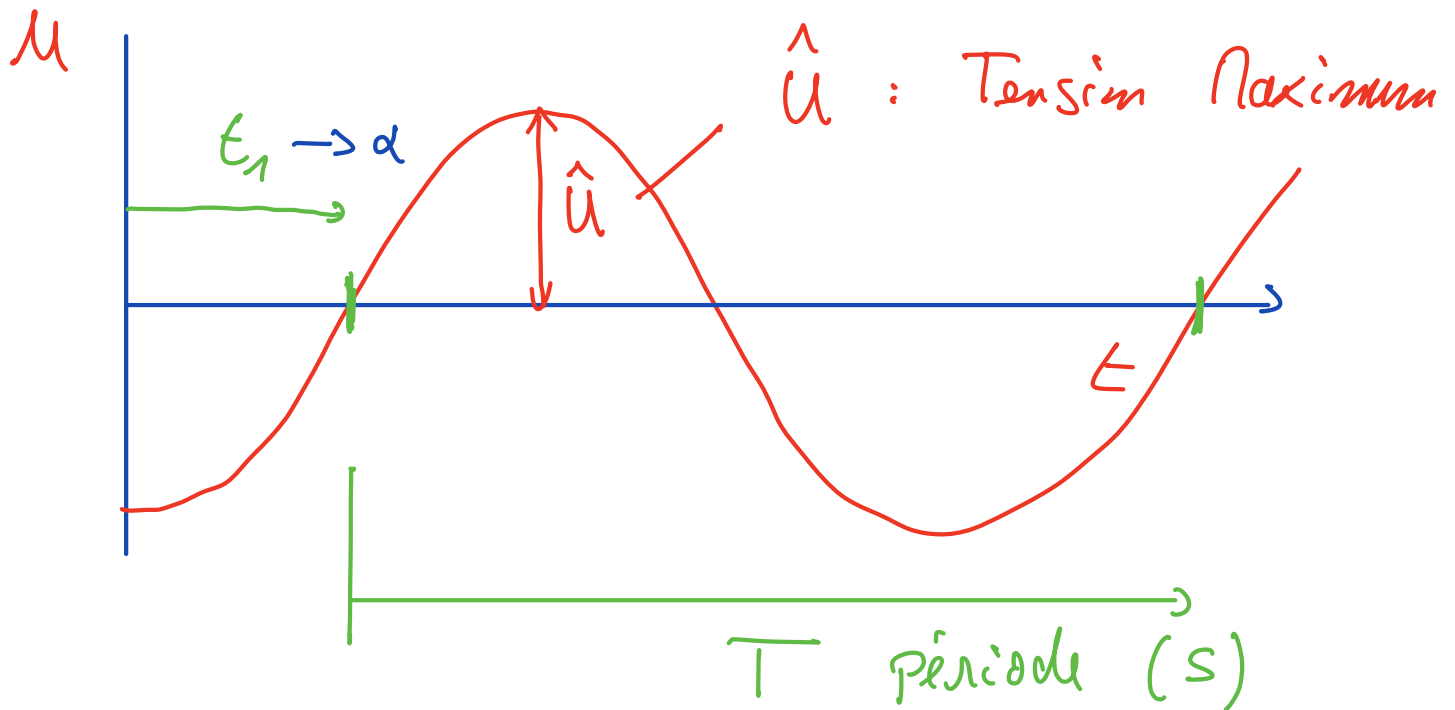
$$\eta = \frac{P_{\text{utile}}}{P_{\text{consommée}}} = \frac{R_L \cdot \cancel{I^2}}{(R_L + R_i) \cancel{I^2}}$$

$$\text{Si } R_i = R_L$$

$$\Rightarrow \eta = 0,5$$

6. Régime Sinusoïdal Permanent :

6.2 Grandeurs Sinusoïdales



$T = \text{période du signal (s)}$

$f = \text{fréquence} = \frac{1}{T} \text{ (Hz)}$

$\omega = \text{pulsation} = \frac{2\pi}{T} = 2\pi \cdot f$

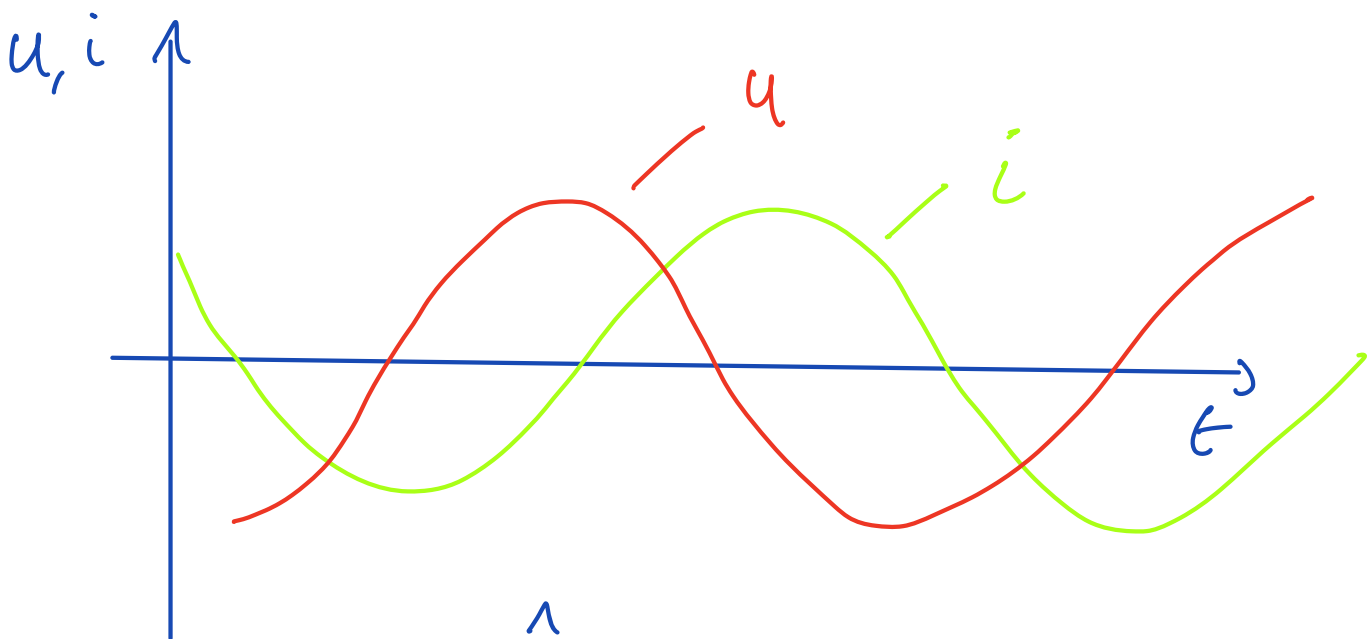
$$\alpha = \frac{t_1 \cdot 2\pi}{T}$$

$$u(t) = \hat{u} \sin(\omega t + \alpha)$$

$$\vdots$$

$$u = \hat{u} \sin(\omega t + \alpha)$$

$\nearrow$
 Pimwcule : $\rightarrow$ grandeur instantanée



$$u = \hat{u} \sin(\omega t + \alpha)$$

$$i = \hat{i} \sin(\omega t + \beta)$$

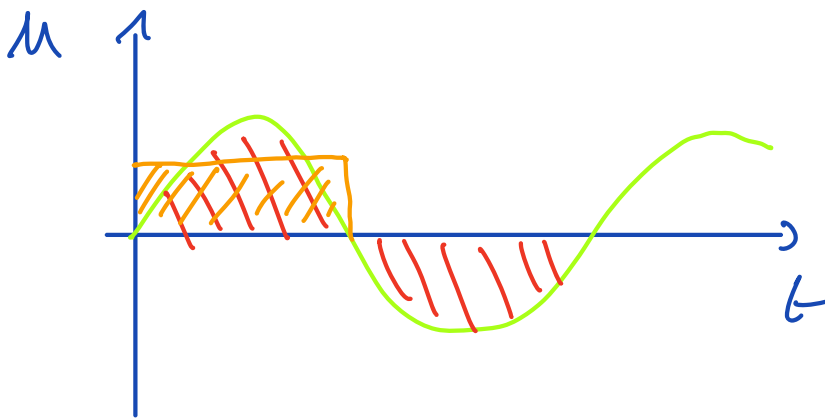
Définition : $\varphi = \alpha - \beta$
 déphasage entre u et i

Définition de la valeur moyenne :

$$\overline{x} = \frac{1}{T} \int_0^T x(t) dt$$

$$\bar{u} = \frac{1}{T} \int_0^T u dt$$

$$\bar{u} = \frac{1}{T} \int_0^T \hat{u} \sin(\omega t + \alpha) dt$$



Prendre sur $T/2$:

$T/2$

$$\bar{u} \Big|_{T/2} = \frac{1}{T/2} \int_0^{T/2} \hat{u} \sin(\omega t) dt \quad \alpha=0$$

$$\bar{u} \Big|_{T/2} = \frac{1}{\omega} \frac{2\hat{u}}{T} \left[-\cos\left(\omega \cdot \frac{T}{2}\right) + \cos(0) \right]$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{\cancel{2} \hat{u} \cancel{T}}{\cancel{T} \cdot \cancel{2} \cdot \pi} \left[-\cos\left(\frac{\cancel{T}}{\cancel{2}} \frac{\cancel{2\pi}}{\cancel{T}}\right) + \cos(0) \right]$$

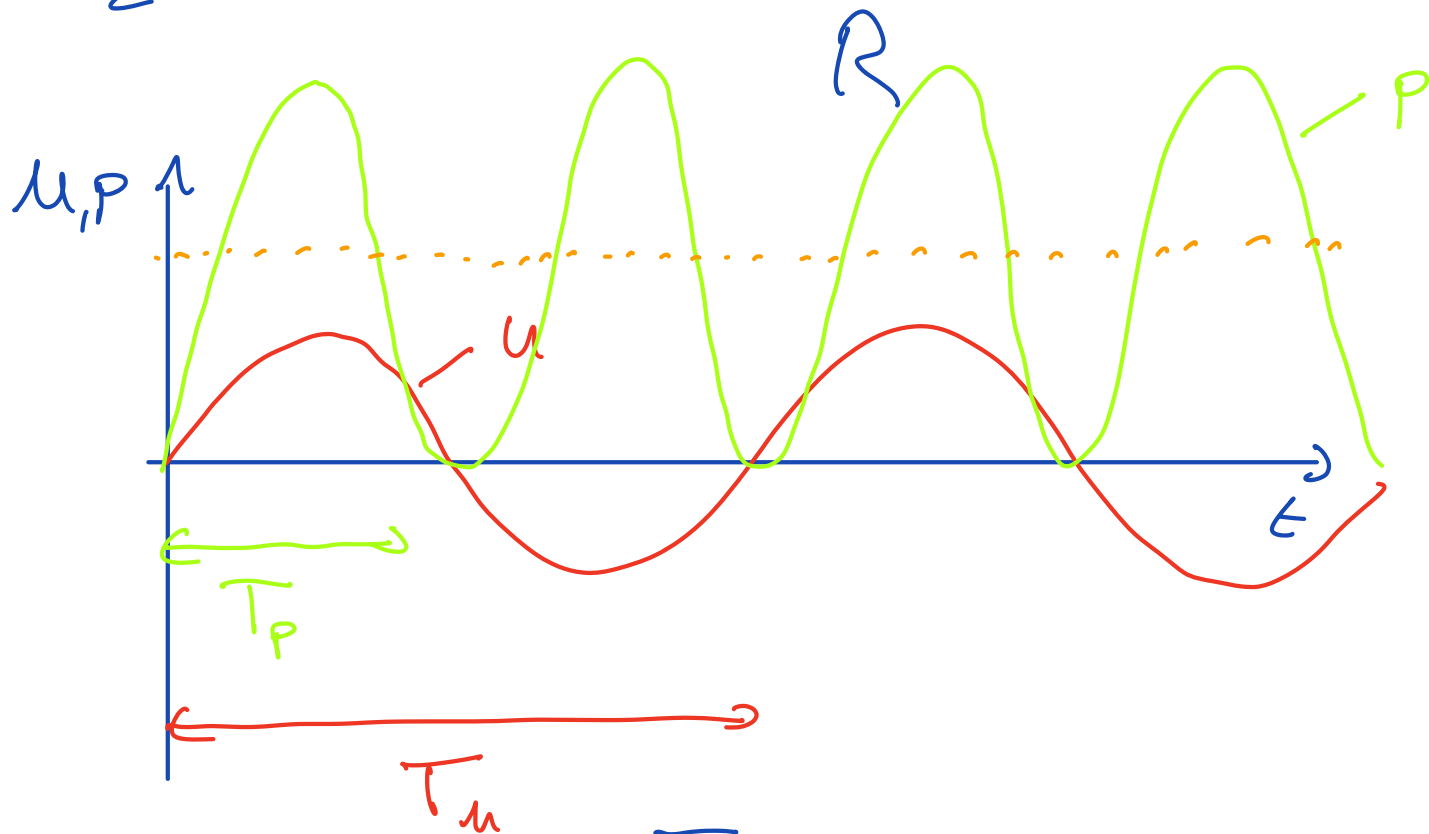
$$= \frac{\hat{u}}{\pi} \left[1 + 1 \right] = \frac{2}{\pi} \hat{u}$$

6.2.13 Puissance instantanée

$$p = u \cdot i$$

$$p = R \cdot i^2 = \frac{u^2}{R}$$

$$\frac{\sin^2 x}{1 - \cos 2x} = \frac{1}{2} \quad \frac{\hat{u}^2 \sin^2(\omega t + \alpha)}{R}$$



$$\bar{p}_R = \frac{1}{T} \int_0^T \frac{\hat{u}^2}{R} \sin^2(\omega t + \alpha) dt$$

$$\bar{p}_R = \frac{1}{T} \int_0^T \frac{\hat{u}^2}{R} \cos^2(\omega t + \alpha) dt$$

$$2 \bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{u}^2}{R} \cdot 1 dt$$

$$2 \bar{P}_R = \frac{\hat{u}^2}{R} \rightarrow \bar{P}_R = \frac{\hat{u}^2}{2R}$$

En alternatif : $\bar{P}_R = \frac{\hat{u}^2}{2R}$

En continu : $P_R = \frac{u^2}{R}$

6.2.12 Valeur efficace :

(RMS Root Mean Square)

Def : $u = \sqrt{\frac{1}{T} \int_0^T \hat{u}^2 \sin^2(\omega t + \alpha) dt}$

↑
Rajusukh

| $\sin^2 x$

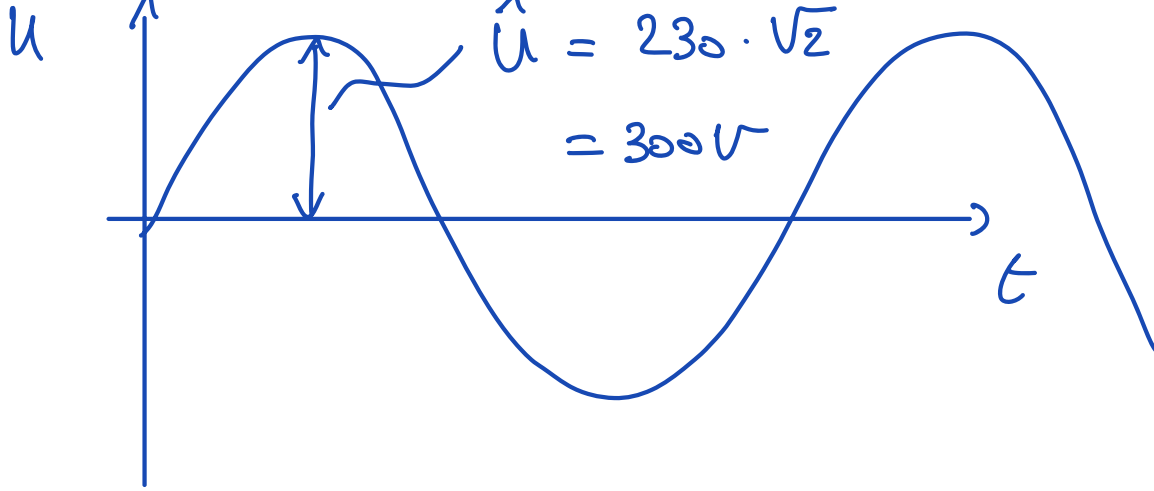
$$= \frac{1 - \cos 2\alpha}{2}$$

$$= \sqrt{\frac{\hat{u}^2}{T} \int_0^T \frac{1}{2} dt - \frac{1}{2} \frac{\hat{u}^2}{T} \underbrace{\int_0^T \cos(2\omega t - 2\alpha) dt}_0}$$

$$u = \frac{\hat{u}}{\sqrt{2}}$$

$$\begin{array}{ccccc} \hat{u} & = & u & \cdot & \sqrt{2} \\ \uparrow & & \uparrow & & \uparrow \\ \text{crête} & & \text{efficace} & & \text{coef. avec sinus} \end{array}$$

$$P_R = \frac{\hat{u}^2}{2R} = \frac{u^2}{R}$$



u, i valeurs instantanées

U, I valeurs efficaces

$\hat{u}, \hat{i}$ valeurs crêtes

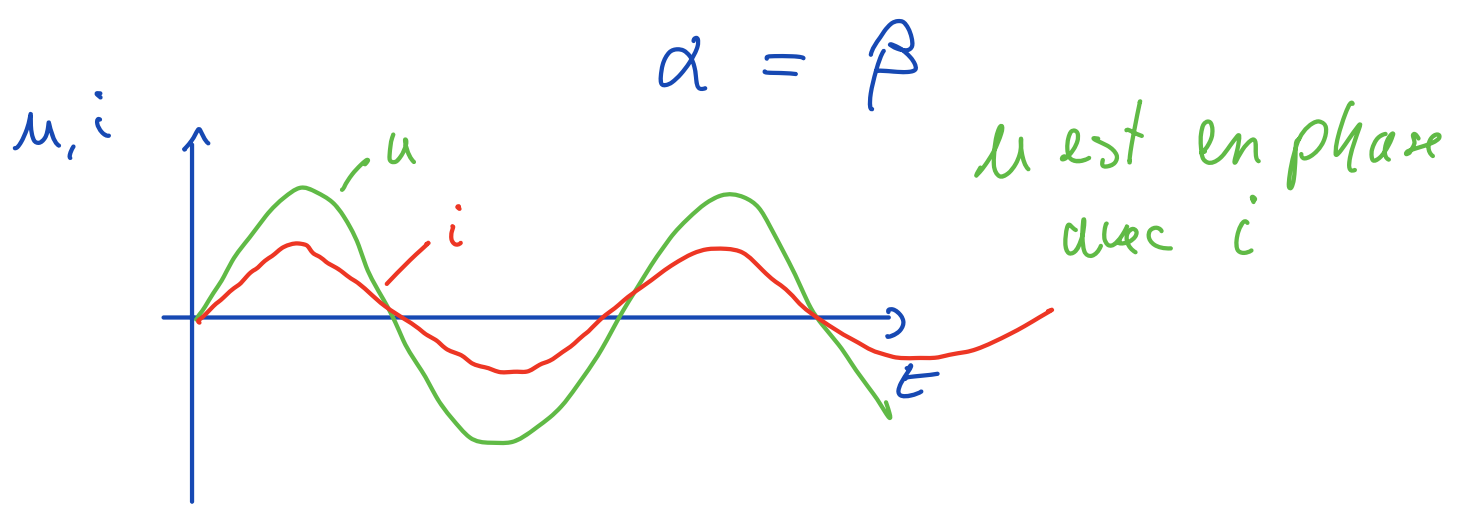
$\bar{u}, \bar{i}$ valeurs moyennes

6.2.14 Cas de R

$$u = R \cdot i$$

$$\hat{u} \cos(\omega t + \alpha) = R \cdot \hat{i} \cos(\omega t + \beta)$$

$$\text{Donc : } \hat{u} = R \cdot \hat{i}$$



6.2.15 cas de L

$$u = L \frac{di}{dt}$$

$$\hat{u} \cos(\omega t + \alpha) = -\omega L \hat{I} \sin(\omega t + \beta)$$

$$= \omega L \hat{I} \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

$$\hat{u} = \omega L \hat{I}$$

$$\alpha = \beta + \frac{\pi}{2}$$

Tension et le courant sont quadrature

retard du courant de $\frac{\pi}{2}$ sur la tension.

6.2.16 cas de C

$$i = C \frac{du}{dt}$$

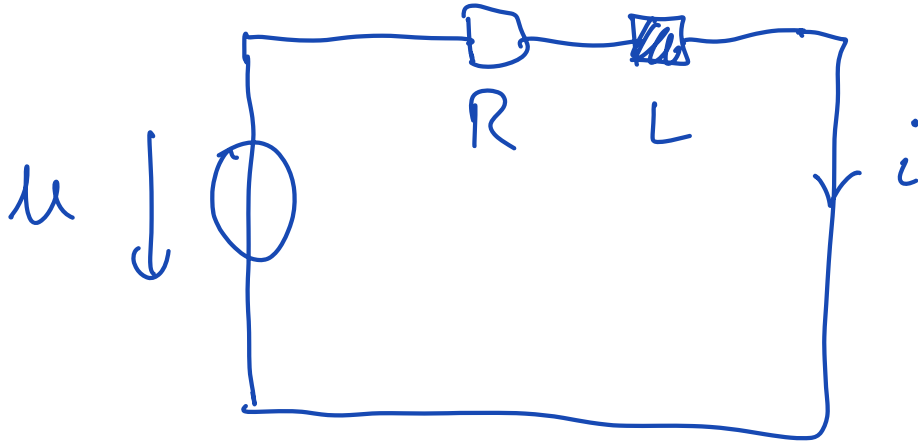
$$\begin{aligned} \hat{I} \cos(\omega t + \beta) &= -\omega C \hat{U} \sin(\omega t + \alpha) \\ &= \omega C \hat{U} \cos\left(\omega t + \alpha + \frac{\pi}{2}\right) \end{aligned}$$

$$\hat{U} = \frac{\hat{I}}{\omega C}$$

$$\alpha = \beta - \frac{\pi}{2}$$

courant en avance
de $\pi/2$ sur la
Tension

6.3 calcul complexe associé :



$$u = u_R + u_L$$

$$u = \hat{u} \sin(\omega t + \alpha) \text{ common}$$

$$i = \hat{i} \sin(\omega t + \beta) \text{ uncommon}$$

→ simplification: $\alpha = 0$

$$\hat{u} \sin(\omega t) = R \cdot \hat{i} \sin(\omega t + \beta) +$$

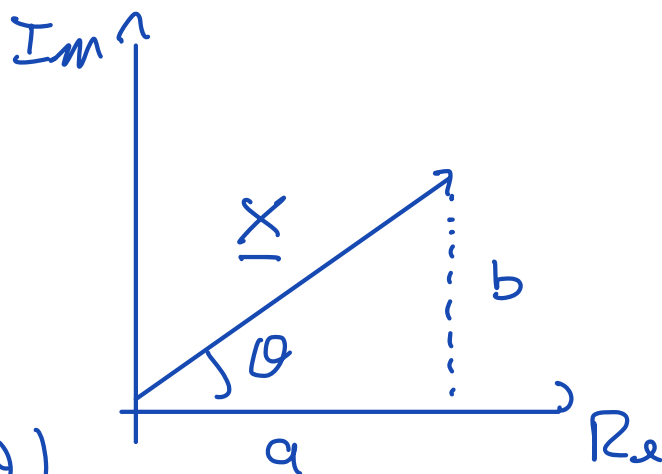
$$\omega L \hat{i} \cos(\omega t + \beta)$$

⋮

complicated !!

Rappel : $j = \sqrt{-1}$

$$\underline{x} = a + bj$$



$$= \hat{x} (\cos \theta + j \sin \theta)$$

$$= \hat{x} e^{j\theta}$$

Concept :

$$\begin{array}{ccc} \mu = \hat{\mu} \sin(\omega t) & \xrightarrow{\text{Tras. compl.}} & \underline{\mu} = \hat{\mu} e^{j(\omega t)} \\ \text{Reel} & & \text{imaginaire} \end{array}$$

$$\mu = \text{Im} \{ \underline{\mu} \}$$

←

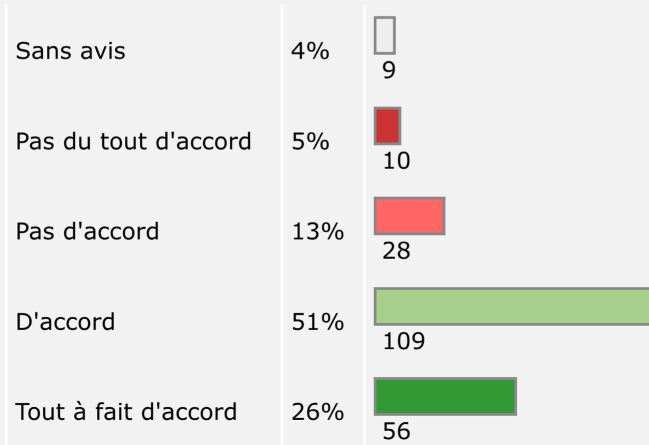
$$\hat{i} = \hat{I} \sin(\omega t + \beta) \longrightarrow \underline{i} = \hat{I} e^{j(\omega t + \beta)}$$

←

$$i = \bigcup_m \{ \underline{i} \}$$

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Questionnaire	 Retour indicatif des enseignements (dès 2022-2023)
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Nb Répondu	212

Le déroulement du cours permet ma formation et un climat de classe approprié



$$u = \hat{U} \cos(\omega t + \alpha)$$

résultats

nb complexes

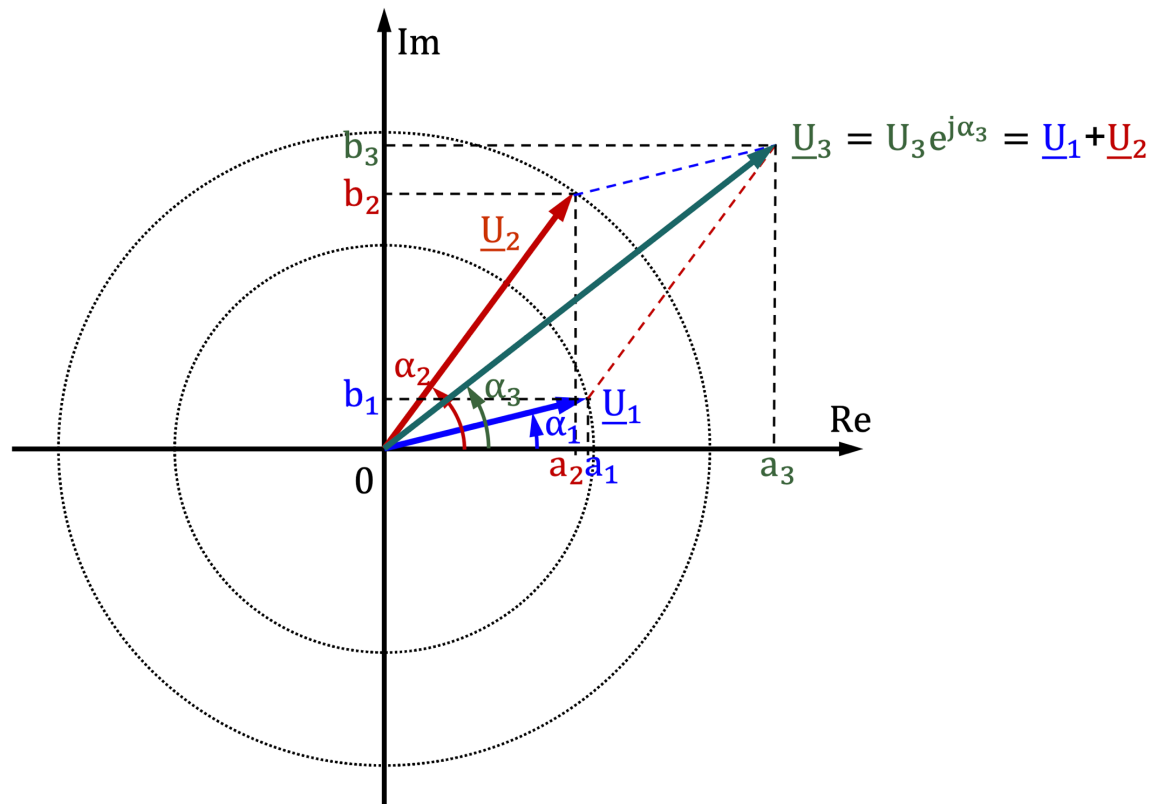
$$\underline{u} = \hat{U} e^{j(\omega t + \alpha)}$$

on fait les
calculs

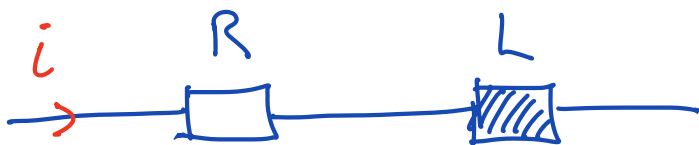
résultats complexes

$\text{Re}\{\dots\}$

Diagramme des phaseurs



Exemple :

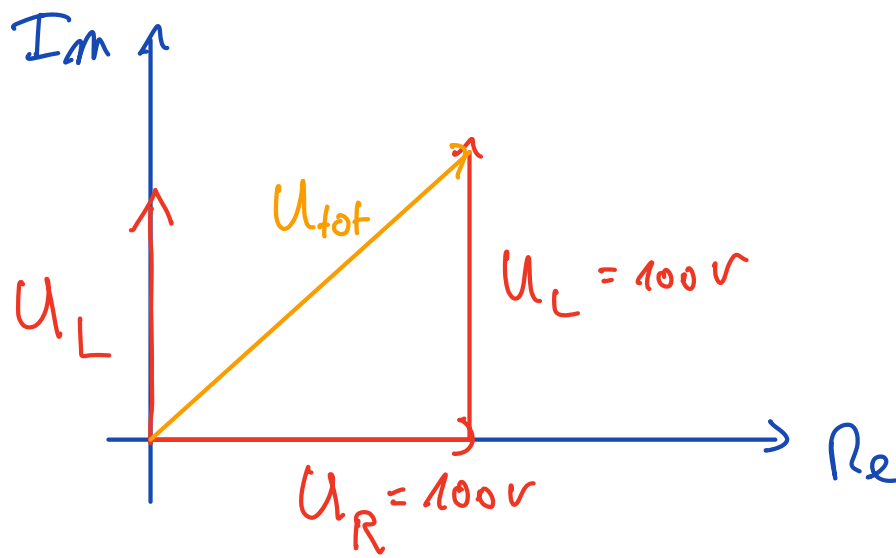


$$U_R = 100\text{V} \quad U_L = 100\text{V}$$



$$U_{\text{tot}} = ?$$

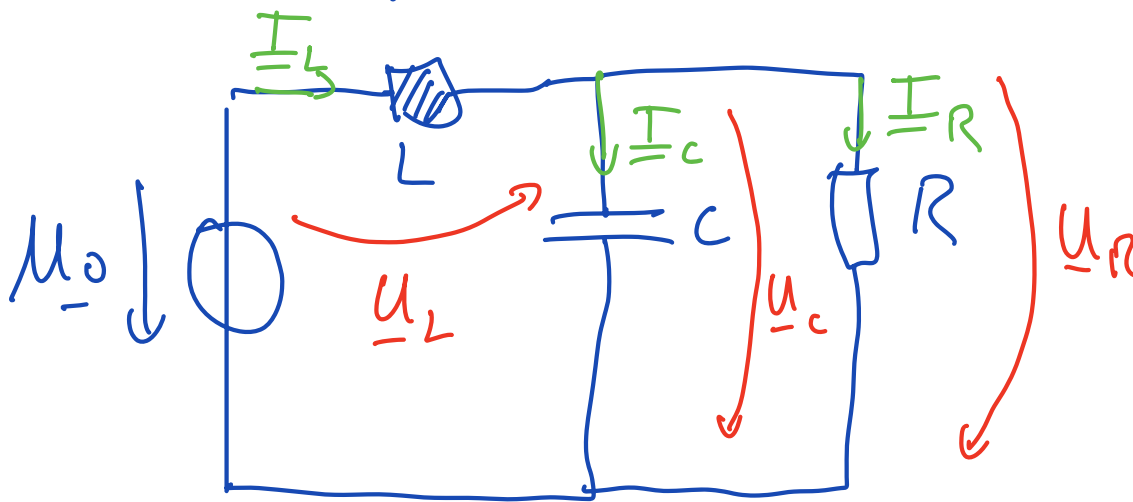
$$u = L \frac{di}{dt}$$



$$\underline{U}_L = j\omega L \underline{I}$$

$$|\underline{U}_{tot}| = \sqrt{U_R^2 + U_L^2} = 141V$$

2^{im} Example:



$$\underline{I}_C = \text{const} \quad \underline{U}_0 ?$$

$$\underline{U}_C = \underline{Z}_C \cdot \underline{I}_C = \frac{1}{j\omega C} \cdot \underline{I}_C = -\frac{j}{\omega C} \cdot \underline{I}_C$$

$$\frac{1}{j\omega C} \cdot \frac{1}{j} = -\frac{j}{\omega C}$$

$$\underline{u}_c = \underline{u}_R \quad \underline{u}_R = R \cdot \underline{I}_R$$

$$\underline{I}_R = \frac{\underline{u}_c}{R} = -\frac{j}{\omega C R} \cdot \underline{I}_c$$

$$\underline{I}_L = \underline{I}_R + \underline{I}_c$$

$$\underline{I}_L = \underline{I}_c \left(1 - j \frac{1}{\omega C R} \right)$$

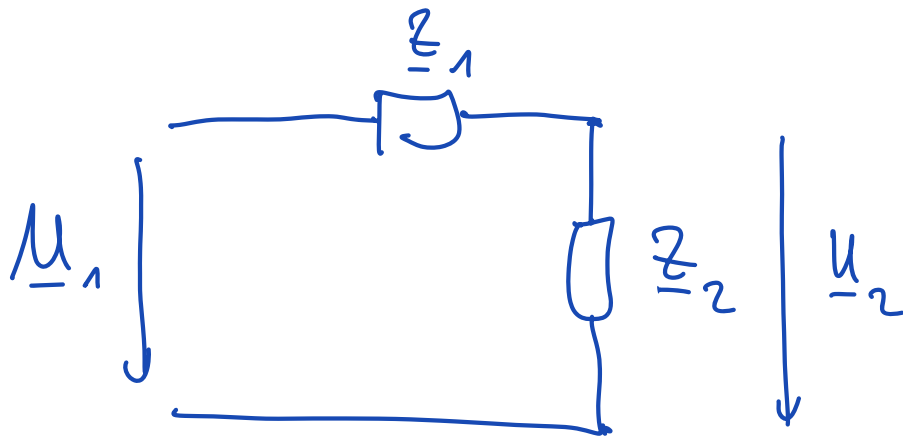
$$\underline{u}_L = \underline{z}_L \cdot \underline{I}_L = j\omega L \underline{I}_L =$$

$$\underline{I}_c \left(\frac{L}{RC} + j\omega L \right)$$

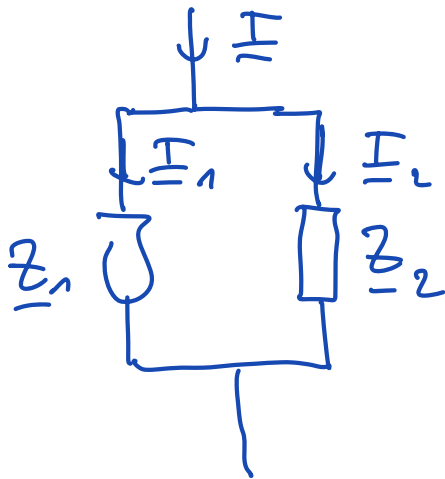
$$\underline{u}_o = \underline{u}_L + \underline{u}_c = \underline{I}_c \left(\frac{L}{RC} + j\omega L \right) - j \frac{1}{\omega C} \underline{I}_c$$

$$= \underline{I}_c \left[\frac{L}{RC} + j \left(\omega L - \frac{1}{\omega C} \right) \right]$$

7.2.5 Diviseur de tension et courant:



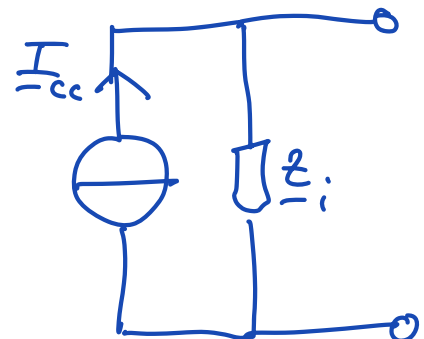
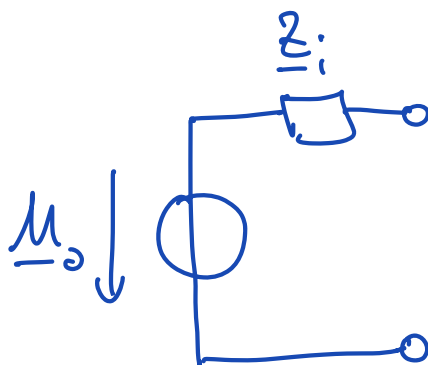
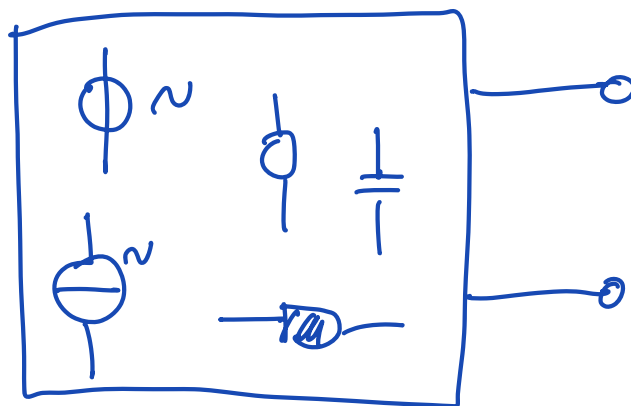
$$\underline{U}_2 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \cdot \underline{U}_1$$



$$\underline{I}_2 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \cdot \underline{I}$$

7.3.1 Théorèmes de Thévenin et Norton

Dipôle :



$\underline{U}_o$ = Tension à vide du circuit

$\underline{I}_{cc}$ = courant de court-circuit

$$\underline{Z}_i = \frac{\underline{U}_o}{\underline{I}_{cc}}$$

Sources de même fréquence

7.4 Principe de Superposition :

Systeme doit être linéaire

Cas No 1 Toutes les sources ont la même fréquence

On considère chaque source séparément :
en annulant les autres :

Source No 1 : $\rightarrow \underline{I}_1$

Source No 2 : $\rightarrow \underline{I}_2$

$\vdots$

$$\underline{I}_{tot} = \sum_{j=1}^K \underline{I}_j \quad K = \text{nb de source}$$

valable pour U et I

cas No 2 : Les sources n'ont pas la même fréquence !

$$f_1 : \rightarrow \underline{I}_{tot 1}$$

$$f_2 : \rightarrow \underline{I}_{tot 2}$$

$\vdots$

$$\underline{I}_{tot 1} \Rightarrow \underline{\dot{I}}_{tot 1} = \sqrt{2} I_{tot 1} e^{j(\omega_1 t + \beta_1)}$$

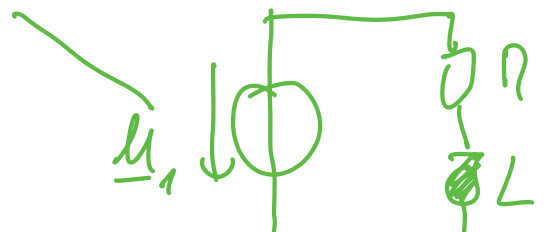
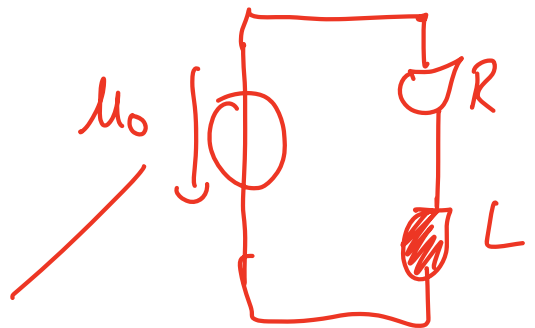
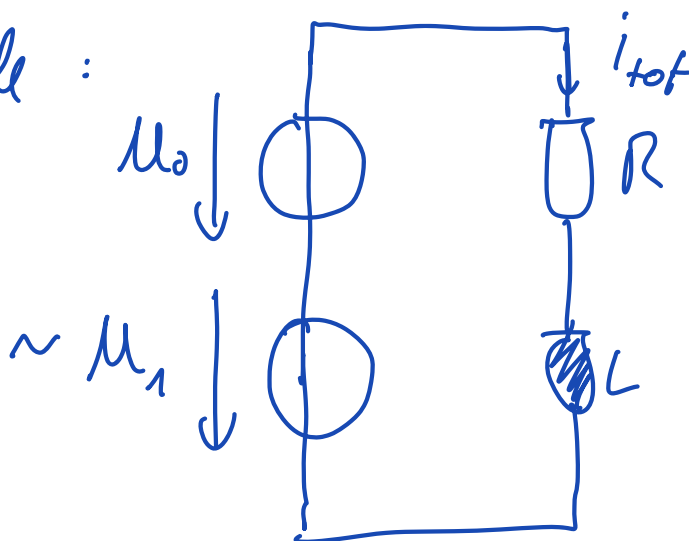
$$\underline{I}_{tot 2} \Rightarrow \underline{\dot{I}}_{tot 2} = \sqrt{2} I_{tot 2} e^{j(\omega_2 t + \beta_2)}$$

$\vdots$

$$\underline{\dot{I}}_{tot} = \underline{\dot{I}}_{tot 1} + \underline{\dot{I}}_{tot 2} \quad (\text{Nonde temporel})$$

$$\underline{\dot{I}}_{tot} = \sqrt{2} I_{tot 1} \sin(\omega_1 t + \beta_1) + \sqrt{2} I_{tot 2} \sin(\omega_2 t + \beta_2)$$

Exemple :

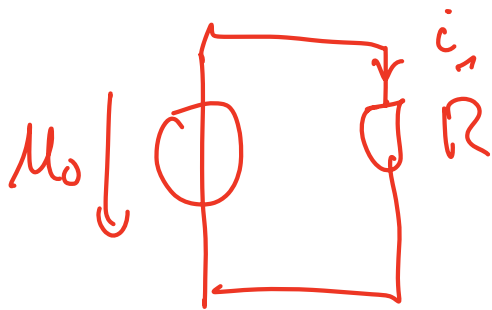


Cas de la source continue U_0 :

$$f = 0 \quad \omega = 0$$

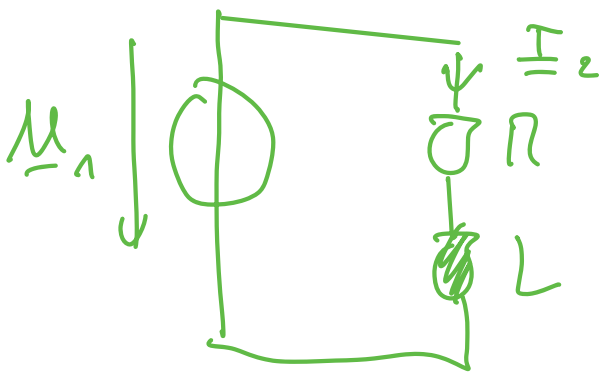
$$\underline{Z}_R = R$$

$$\underline{Z}_L = j\omega L = 0$$



$$i_1 = \frac{U_0}{R}$$

Cas de la source U_1 :



$$\underline{Z}_R = R$$

$$\underline{Z}_L = j\omega L$$

$$\underline{Z}_{tot} = R + j\omega L$$

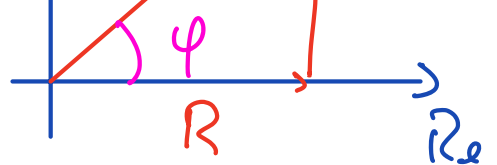
$$U_1 = \underline{Z}_{tot} \cdot \underline{I}_2$$

$$|\underline{Z}_{tot}| = \sqrt{R^2 + \omega^2 L^2}$$

$I_n \uparrow$



$$\varphi = \arctan \frac{\omega L}{R}$$



$$\underline{u}_1 = u_1 e^{j^{(0)}} \quad (\text{on pose } \alpha = 0)$$

$$= u_1$$

$$\underline{I}_2 = \frac{\underline{u}_1}{\underline{Z}_{\text{tot}}} = \frac{u_1}{\sqrt{R^2 + \omega^2 L^2}} \frac{e^{j^0}}{e^{j\varphi}} = I_2 e^{j-\varphi}$$

$$\underline{i}_2 = \sqrt{2} I_2 e^{j(\omega t - \varphi)} \Rightarrow j(\omega t - \varphi)$$

Notion de réel : $i_2 = \sqrt{2} I_2 \sin(\omega t - \varphi)$

Résultat final : $i_{\text{tot}} = \frac{u_0}{R} + \sqrt{2} I_2 \sin(\omega t - \varphi)$

8. Puissances en alternatif sinus

Nonohmique:

8.1 Puissance instantanée:

$$P(t) = p = u \cdot i$$

$$u = \hat{u} \cos(\omega t + \alpha)$$

$$i = \hat{I} \cos(\omega t + \beta)$$

$$p = \hat{u} \hat{I} \cos(\omega t + \alpha) \cos(\omega t + \beta)$$

$$\cos x \cdot \cos y = \frac{1}{2} \left(\cos(x-y) + \cos(x+y) \right)$$

$$p = \frac{\hat{u} \hat{I}}{2} \left[\underbrace{\cos(\alpha - \beta)}_{\varphi} + \cos(2\omega t + \alpha + \beta) \right]$$

$$p = u \cdot I \left[\cos \varphi + \cos(2\omega t + \alpha + \beta) \right]$$

